The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 2 – Reason abstractly and quantitatively.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 2.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

Overview of MHM2 – What it is, What it does, and What it looks like

MHM2. Reason abstractly and quantitatively.			
What it is	**What it does**	**What it looks like**	
Breaking apart a problem and showing it symbolically, with pictures, or in any way other than the standard algorithm.	Allows students to independently figure out what to do within the context of a problem. • Make sense of quantities and the relationships between quantities, in problem situations. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and flexibly use operations and their properties. • Create a logical representation of the problem. • Attend to the meaning of quantities, not just how to compute them.	***Students:*** • Use varied representations and approaches when solving problems. • Make sense of quantities and their relationships in problem situations. • Are decontextualizing and contextualizing. • Use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, not just how to compute them.	***Teacher:*** • Provides a range of representations of math problem situations and encourages various solutions. • Provides opportunities for students to make sense of quantities and their relationships in problem situations. • Provides problems that require flexible use of properties of operations and objects. • Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them and/or rules; and knowing and flexibly using different properties of operations and objects.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM2. Reason abstractly and quantitatively.	I can think about the math problem. I can make sense of the numbers in the problem and how they are related to each other. I can create an equation or equations to represent the problem. I can use units to help me understand what the numbers mean.	• How do you know? • What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____ related to _____? • What is the relationship between _____ and _____? • What does _____ mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide that you needed to use _____ in this task? • Could we have used another operation or property to solve this task? Why or why not?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM2	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Reason abstractly and quantitatively.	**Task**	• Lacks context. • Does not make use of multiple representations or solution paths.	• Is embedded in a contrived context.	• Has realistic context. • Requires students to frame solutions in a context. • Has solutions that can be expressed with multiple representations.	• Has relevant realistic context.
	Teacher	• Does not expect students to interpret representations. • Expects students to memorize procedures with no connection to meaning.	• Expects students to model and interpret tasks using a single representation. • Explains connections between procedures and meaning.	• Expects students to interpret and model using multiple representations. • Provides structure for students to connect algebraic procedures to contextual meaning. • Links mathematical solution with a question's answer.	• Expects students to interpret, model, and connect multiple representations. • Prompts students to articulate connections between algebraic procedures and contextual meaning.

The Vertical Progression of the Mathematical Habit of Mind 2

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM2 - Reason abstractly and quantitatively.	
Kindergarten Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create a representation of a problem while attending to the meanings of the quantities. For example: » a student may write the numeral 11 to represent an amount of objects counted, » select the correct number card, 17, to follow 16 on a calendar, or » build two piles of counters to compare the numbers 5 and 8. • begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. • need to answer questions such as "How do you know?"—which reinforces reasoning and understanding and helps develop mathematical language.
Grade 1 Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create a representation of a problem while attending to the meanings of the quantities. • make sense of quantities and relationships while solving tasks. • represent situations by decontextualizing tasks into numbers and symbols. For example: » "There are 14 children on the playground, and some children go line up. If there are 8 children still playing, how many children lined up?" » Students translate the problem into the situation equation 14 − __ = 8, then into the related equation 8 + __ = 14, and then solve the task. • contextualize situations during the problem-solving process. For example: » refer to the context of the task to determine the need to subtract 8 from 14, because the number of children in line is the total number less the 8 who are still playing. • reason about ways to partition two-dimensional geometric figures into halves and fourths.
Grade 2 Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create a representation of a problem while attending to the meanings of the quantities. • represent situations by decontextualizing tasks into numbers and symbols. For example: » a task may be presented as follows: "There are 25 students in the cafeteria, and they are joined by 17 more students. How many students are in the cafeteria?" » Students translate the situation into an equation (such as 25 + 17 = __) and then solve the problem. • contextualize situations during the problem-solving process.

MHM2 - Reason abstractly and quantitatively.	
Grade 3 Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create a logical representation of the problem, considering both the appropriate units involved and the meaning of quantities. • apply understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 × __ = 40, they might think: 4 groups of some number is the same as 40. 4 times some number is the same as 40. 4 groups of 10 is 40, so the unknown number is 10. The missing factor is 10, because 4 times 10 equals 40.
Grade 4 Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create a logical representation of the problem, considering both the appropriate units involved and the meaning of quantities. • extend understanding from whole numbers to fractions and decimals. • write simple expressions, record calculations with numbers, and represent or round numbers using place-value concepts. • use array or area drawings to demonstrate and explain 154 × 6 as 154 added six times and develop an understanding of the distributive property. For example: 154 × 6 = (100 + 50 + 4) × 6 = (100 × 6) + (50 × 6) + (4 × 6) = 600 + 300 + 24 = 924
Grade 5 Students:	• recognize a number represents a specific quantity and connect the quantity to written symbols. • create logical representations of problems, considering appropriate units and the meaning of quantities. • extend understanding from whole numbers to work with fractions and decimals. • write simple expressions, record calculations with numbers, and represent or round numbers using place-value concepts. • use abstract and quantitative thinking to recognize, without calculating the quotient, that 0.5 × (300 ÷ 15) is $\frac{1}{2}$ of (300 ÷ 15).
Grade 6 Students:	• represent a wide variety of real-world contexts by using rational numbers and variables in mathematical expressions, equations, and inequalities. • contextualize to understand the meaning of the number or variable as related to the problem. • decontextualize to operate with symbolic representations by applying properties of operations or other meaningful moves.
Grade 7 Students:	• represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • contextualize to understand the meaning of the number or variable as related to the problem. • decontextualize to manipulate symbolic representations by applying properties of operations.

MHM2 - Reason abstractly and quantitatively.	
Grade 8 Students:	• represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • examine patterns in data. • assess the degree of linearity of functions. • contextualize to understand the meaning of the number(s) or variable(s) related to the problem. • decontextualize to manipulate symbolic representations by applying properties of operations.
Algebra I and Math I Students:	• extend understanding of slope as the rate of change of a linear function. • comprehend the average rate of change of any function can be computed over an appropriate interval.
Geometry and Math II Students:	• understand the coordinate plane can be used to represent geometric shapes and transformations. • connect understanding of number and algebra to geometry.
Algebra II and Math III Students:	• deepen understanding of variables—for example, by understanding changing the values of the parameters in an expression has consequences for the graph of the function. • interpret parameters in a real-world context.