The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 3 – Construct viable arguments and critique the reasoning of others.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 3.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

Overview of MHM3 – What it is, What it does, and What it looks like

MHM3. Construct viable arguments and critique the reasonings of others.			
What it is	**What it does**	**What it looks like**	
Talking about math, using mathematical language to kindly support or oppose the work of others.	Encourages students to participate in mathematical discourse in an environment where they feel safe to discuss their ideas, ask questions, and justify their answers. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve or revise the argument. • Compare two arguments and determine if the logic is correct or flawed.	***Students:*** » Make conjectures and explore the truth of their conjectures. » Recognize and use counterexamples. » Justify and defend ALL conclusions and communicate them to others. » Recognize and explain flaws in arguments. (After listening or reading arguments of others, they respond by deciding whether or not the arguments make sense. They ask useful questions to improve arguments.) » Construct arguments using concrete referents such as objects, drawings, diagrams, or actions. Later, students learn to determine the domains to which an argument applies.	***Teacher:*** » Provides ALL students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments. » Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures. » Provides opportunities for students to construct arguments and critique arguments of peers. » Facilitates and guides students in recognizing and using counterexamples. » Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others. » Asks useful questions to clarify and/or improve students' arguments.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM3. **Construct viable arguments and critique the reasoning of others.**	I can make a plan, called a strategy, to solve the problem and discuss other students' strategies, too.	• What mathematical evidence would support your solution? • How can we be sure that _______? • How could you prove that _______? • Will it still work if _______? • What were you considering when _______? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about _______? • How could you demonstrate a counterexample? • I think it might be clearer if you said _______. Is that what you meant? • Is your method like BLANK's method? If not, how is your method different? • Do you think that would happen all the time? • I wonder why _________?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM3	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Construct viable arguments and critique the reasoning of others.	**Task**	• Is ambiguously stated.	• Is not at the appropriate level.	• Avoids single steps or routine algorithms.	
	Teacher	• Does not ask students to present arguments or solutions. • Expects students to follow a given solution path without opportunities to make conjectures.	• Does not help students differentiate between assumptions and logical conjectures. • Asks students to present arguments but not to evaluate them. • Allows students to make conjectures without justification.	• Identifies students' assumptions. • Models evaluation of student arguments. • Asks students to explain their conjectures.	• Helps students differentiate between assumptions and logical conjectures. • Prompts students to evaluate peer arguments. • Expects students to formally justify the validity of their conjectures.

The Vertical Progression of the Mathematical Habit of Mind 3

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM3 – Construct viable arguments and critique the reasoning of others.	
Kindergarten Students:	• construct arguments using actions and concrete materials, such as objects, pictures, or drawings. • explain thinking to others and respond to others' thinking.
Grade 1 Students:	• construct arguments using concrete referents, such as objects, pictures, drawings, and actions. • explain thinking and listen to the explanations of others. For example: » "There are 9 books on the shelf. » If you put more books on the shelf and there are now 15 books on the shelf, how many books did you put on the shelf?" • use a variety of strategies to solve tasks and then share and discuss problem-solving strategies with classmates.
Grade 2 Students:	• construct arguments using concrete referents, such as objects, pictures, math drawings, and actions. • not only explain thinking, but also listen to others' explanations. • decide if the explanations make sense and ask appropriate questions. • critique the strategies and reasoning of classmates.
Grade 3 Students:	• construct arguments using concrete referents such as objects, pictures, and drawings. • explain thinking to others and respond to others' thinking.
Grade 4 Students:	• construct arguments using concrete referents, such as objects, pictures, drawings, and actions. • not only explain thinking, but also listen to others' explanations and ask questions. • explain and defend answers and solution strategies.
Grade 5 Students:	• construct arguments by using visual models such as objects and drawings. • explain calculations based upon models, properties of operations, and rules that generate patterns. • demonstrate and explain the relationship between volume and multiplication. • explain thinking to others and respond to others' thinking. • use various strategies to solve problems. • defend and justify work.

MHM3 – Construct viable arguments and critique the reasoning of others.	
Grade 6 Students:	• construct arguments with verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). • further refine mathematical communication skills through mathematical discussions. • critically evaluate thinking and the thinking of others. • explain thinking to others and respond to others' thinking.
Grade 7 Students:	• construct arguments with verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables. • further refine mathematical communication skills through mathematical discussions. • critically evaluate thinking and the thinking of others. • construct viable arguments and critique the reasoning of others.
Grade 8 Students:	• construct arguments with verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). • further refine mathematical communication skills through mathematical discussions. • critically evaluate thinking and the thinking of others. • explain thinking to others and respond to others' thinking.
Algebra I and Math I Students:	• reason through the solving of equations. • recognize solving an equation involves more than simply following rote rules and steps. • use language such as "If ________, then ________" when explaining solution methods. • provide justification for reasoning.
Geometry and Math II Students:	• reason through the solving of equations. • recognize solving an equation involves more than simply following rote rules and steps. • use language such as "If ________, then ________" when explaining solution methods. • provide justification for reasoning.
Algebra II and Math III Students:	• continue to reason through the solution of an equation. • justify reasoning to peers. • defend choice of a function when modeling a real-world situation.