West Virginia
College & Career
Readiness Standards

Support for College- and Career-Readiness Standards

PERSONALIZED MATHEMATICS THIRD & FOURTH COURSE OPTIONS

Middle and Secondary Services
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Introduction

Support for College- and Career-Readiness Standards: Personalized Mathematics Third and Fourth Course Options is a companion document to the *West Virginia College- and Career-Readiness Standards.*

This document prepares students for study of the grade-level standards through the teaching of prerequisite and enabling skills necessary for mastering each standard. **This allows students to work toward grade-level and course content standards while working at individual ability levels.** By identifying the prerequisite and enabling skills for each standard, teachers may plan instruction **to address the achievement gap.**

Support for College- and Career-Readiness Standards: Personalized Mathematics Third and Fourth Course Options provides short-term objectives to help students reach grade-level standards. Educators are encouraged to use the support document to:

- Write annual Individualized Education Program (IEP) goals
- Design targeted interventions
- Write learner objectives
- Develop lesson plans
- Plan for instructional grouping
- Plan for parent communication and conferences
- Prepare students for mastery of state standards

This document helps educators recognize what students are able to do in relation to the course-specific standards for Advanced Mathematical Modeling, Applied Statistics, Financial Algebra/ Mathematics, High School Mathematics IV-Trigonometry/Pre-calculus, Technical Transition Mathematics for Seniors and Transition Mathematics for Seniors in order to help them move toward explicit success criteria.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local education agencies (LEAs) may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Navigating the Document

The West Virginia College-and Career-Readiness Standards (WVCCRS) will be listed by domain on the left- hand side of each page. The right-hand column labeled *"Can this student…?"* lists suggested supports, or steps, that a student might need to accomplish prior to mastering the grade-level standard.

The bullet at the top of this list is generally the skill closest to the course-specific standard. If a student is unprepared to demonstrate the skill at the top of the list, the teacher should continue to move down the list of suggestions until he or she identifies what that student is able to do. The bulleted list can be thought of as a ladder; starting at the bottom, educators help the student step up each rung until they are ready to demonstrate mastery of the grade-level standard. It is important to remember the pre-requisite skills are **not meant to replace** the grade-level standard nor are they a "break-down" of the standard itself.

For example, if a student has not yet mastered WVCCRS M.6.28, "*Display numerical data in plots on a number line, including dot plots, histograms and plots,*" the teacher should look to the first bullet listed to the right under "Can this student…?" The teacher would ask "*Can this student organize, display, and interpret information in line plot with a horizontal scale in fractional units?*" If not, move to the next bullet: "*Can this student calculate median, mean, and range for a data set?*" Continue down the list until the teacher finds an appropriate starting point for instruction.

Cluster	**Summarize and describe distributions.**	**Can this student…?**
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.	• Organize, display, and interpret information in line plots with a horizontal scale in fractional units • Calculate median, mean, and range for a data set • Create number lines using appropriate intervals

*Note for educators writing IEP goals:

Goals should address student's unique needs across the content areas and should link to the West Virginia College- and Career-Readiness Standards so that a student has the foundation or precursor skills and strategies needed to access and progress in the general education curriculum. Keep in mind that the standards themselves are generally not stated in measurable terms and cannot be substituted for individually developed goals. Rather, the annual goal should focus on what is needed for the student to learn and attain the grade-level standard.

Support for Mathematics Standards

Advanced Mathematical Modeling

All West Virginia teachers are responsible for classroom instruction that integrates content standards and Mathematical Habits of Mind. Primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision-making in finance, and spatial and geometric modeling for decision-making. Students will learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As students solve problems in various applied situations, they will develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they will rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students will use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Developing College and Career Skills	**Finance**
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**
• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and	• Solve geometric problems involving

consider the advantages/disadvantages of each method.	inaccessible distances. • Use vectors to solve applied problems.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Developing College and Career Skills	
Math as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Finance	
Understanding financial models.	Standards 4-6
Personal use of finance.	Standards 7-8
Probability	
Analyzing information using probability and counting.	Standards 9-10
Managing uncertainty.	Standards 11-12
Statistics	
Critiquing statistics.	Standards 13-16
Conducting statistical analysis.	Standards 17-21
Communicating statistical information.	Standards 22-23
Modeling	
Managing numerical data.	Standards 24-25
Modeling data and change with functions.	Standards 26-30
Networks	
Networking for decision making.	Standards 31-32
Social Decision Making	
Making decisions using ranking and voting.	Standards 33-34
Geometry	
Concrete geometric representation (physical modeling).	Standards 35-36
Abstract geometric representation (matrix modeling).	Standards 37-38

Developing College and Career Skills

Cluster	Math as a language	Can this student…?
M.AMM.1	Demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.	• Solve real-world and mathematical problems by writing and solving equations • Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related
M.AMM.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.	• Organize, display, and interpret information in tables and graphs (frequency tables, pictographs, and line plots) • Compare representations of data; including graphs, tables, equations; within the context of the data • Solve real-world and mathematical problems by writing and solving equations • Identify important quantities in a practical situation • Describe qualitatively the functional relationship between two quantities by analyzing the graph (e.g., increasing or decreasing, linear or nonlinear, etc.)
Cluster	**Tools for problem solving**	**Can this student…?**
M.AMM.3	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.	• Recognize the purposes and differences among sample surveys, experiments, and observational studies • Make inferences and justify conclusions from sample surveys, experiments, and observational studies • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Organize, display, and interpret information in various graphs (e.g., scatter plots and line graphs) • Explain how to draw inferences from data distributions

Finance

Cluster	Understanding financial models	Can this student…?
M.AMM.4	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Auto, Mortgage, and/or Credit Card).	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of data including graphs, tables, equations, and context • Write a function that describes a relationship between two quantities
M.AMM.5	Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit.)	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of data including graphs, tables, equations, and context • Write a function that describes a relationship between two quantities
M.AMM.6	Determine, represent, and analyze mathematical models for Inflation and the Consumer Price Index using concepts of rate of change and percentage growth.	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of data including graphs, tables, equations, and context • Write a function that describes a relationship between two quantities
Cluster	**Personal Use of Finance**	**Can this student…?**
M.AMM.7	Research and analyze personal budgets based on given parameters (e.g., Fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement and/or investment planning, etc.).	• Solve real-world and mathematical problems by writing and solving equations • Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies • Understand that positive and negative numbers are used together to describe quantities having opposite directions or values
M.AMM.8	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.	• Solve real-world and mathematical problems by writing and solving equations • Apply properties of operations to calculate with

		numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies • Use proportional relationships to solve multistep ratio and percent problems • Understand that positive and negative numbers are used together to describe quantities having opposite directions or values

Probability

Cluster	**Analyzing information using probability and counting**	**Can this student…?**
M.AMM.9	Use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.	• Use permutations and combinations to compute probabilities of compound events and solve problems • Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Know and use factorial notation
M.AMM.10	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these ideas with long-run frequency • Define conditional probability and independence • Determine the probability of a simple event
Cluster	**Managing uncertainty**	**Can this student…?**
M.AMM.11	Use probabilities to make and justify decisions about risks in everyday life.	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these ideas with long-run frequency • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and then predict the approximate relative frequency given the probability • Calculate and compare with rational numbers • Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Determine the probability of a simple event

M.AMM.12	Calculate expected value to analyze mathematical fairness, payoff and risk.	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these concepts with long-run frequency • Analyze both costs and benefits in different contextual situations • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and then predict the approximate relative frequency given the probability • Calculate and compare with rational numbers

Statistics

Cluster	Critiquing statistics	Can this student…?
M.AMM.13	Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences
M.AMM.14	Interpret and compare the results of polls, given a margin of error.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
M.AMM.15	Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.	• Distinguish between correlation and causation • Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
M.AMM.16	Describe strengths and weaknesses of sampling techniques, data and graphical	• Understand that statistics can be used to

	displays and interpretations of summary statistics and other results appearing in a study, including reports published in the media.	gain information about a population by examining a sample of the population • Organize, display, and interpret information in various graphs • Understand that random sampling tends to produce representative samples and support valid inferences • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association
Cluster	**Conducting statistical analysis**	**Can this student…?**
M.AMM.17	Identify the population of interest, select an appropriate sampling technique and collect data.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability
M.AMM.18	Identify the variables to be used in a study.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
M.AMM.19	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.	• Understand that random sampling tends to produce representative samples and support valid inferences

		• Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association
M.AMM.20	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features.	• Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the context of the data sets accounting for possible effects of extreme data points (outliers) • Organize, display, and interpret information in various graphs • Find the mean and standard deviation of a data set using technology
M.AMM.21	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.	• Recognize that a measure of center for a numerical data set summarizes all of its values with a single number • Identify mean, median, mode, and range and explain how to calculate each • Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be describe by its center, spread, and overall shape
Cluster	**Communicating Statistical Information**	**Can this student…?**
M.AMM.22	Report results of statistical studies to a particular audience, including selecting an appropriate	• Understand that statistics can be used to gain information about a population by examining a sample of the population

	presentation format, creating graphical data displays and interpreting results in terms of the question studied.	• Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)
M.AMM.23	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)

Modeling

Cluster	**Managing numerical data**	**Can this student…?**
M.AMM.24	Solve problems involving large quantities that are not easily measured.	• Perform operations with numbers expressed in scientific notation
M.AMM.25	Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.	• Use matrices to represent and manipulate data • Interpret expressions that represent a quantity in terms of its context • Identify important quantities in a practical situation • Use substitution to verify algebraically the accuracy of the solution
Cluster	**Modeling data and change with functions**	**Can this student…?**
M.AMM.26	Determine or analyze an appropriate model for problem situations – including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale).	• Graph a linear, logarithmic, or exponential function • Read and interpret information from a graph or table • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is linear or nonlinear) • Interpret rate of change in the context of a real-world situation
M.AMM.27	Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights, diurnal cycle, and/or music).	• Identify amplitude, frequency, and midline • Identify a function that models a relationship between two quantities and interpret key features of graphs • Use properties of right triangles to solve problems

M.AMM.28	Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates).	• Use substitution of input values to determine output values of square root, cube root, piecewise, and absolute value equations • Graph a linear or quadratic equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify a function that models a relationship between two quantities and interpret key features of graphs • Factor a quadratic expression to reveal the zeros of the function it defines
M.AMM.29	Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay).	• Recognize that sequences are functions, sometimes defined recursively • Determine an explicit expression, a recursive process, or steps for calculation from a context
M.AMM.30	Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.	• Organize, display, and interpret information in various graphs • Sketch a line of best fit for a set of data points • Determine if data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data

Networks

Cluster	Networking for decision making	Can this student…?
M.AMM.31	Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, Kolam drawings of India, traveling salesman problem, and/or map coloring).	• Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation
M.AMM.32	Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures.	• Write a flow proof

Social Decision Making

Cluster	Making decisions using ranking and voting	Can this student…?
M.AMM.33	Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results).	• Organize and compare data sets • Develop a probability model and use it to find probabilities of events
M.AMM.34	Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting).	• Organize and compare data sets • Develop a probability model and use it to find probabilities of events • Calculate and compare with rational numbers

Geometry

Cluster	Concrete geometric representation (physical modeling)	Can this student…?
M.AMM.35	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.	• Apply geometric methods to solve design problems • Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation • Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Describe the two-dimensional figures that result from slicing a three-dimensional figure • Represent three-dimensional figures using nets made up of rectangles and triangles
M.AMM.36	Solve geometric problems involving inaccessible distances.	• Use properties of right triangles to solve problems • Use informal arguments to establish facts about the angle sum in triangles

Cluster	Abstract geometric representation (matrix modeling)	Can this student…?
M.AMM.37	Use vectors to represent and solve applied problems.	• Use properties of right triangles to solve problems • Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two- and three-dimensions • Apply the Pythagorean theorem to find the distance between two points in a coordinate system
M.AMM.38	Use matrices to represent geometric transformations and solve applied problems.	• Use matrices to represent and manipulate data • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them

Support for Mathematics Standards

Applied Statistics

All West Virginia teachers are responsible for classroom instruction that integrates content standards and Mathematical Habits of Mind. Applied Statistics provides authentic experiences in statistics designed to strengthen students' application of the statistical method. Students will conduct statistical simulations to model everyday situations in an increasingly data-rich world. Students in this course will select appropriate graphical and numerical methods to explore data, design and implement a plan to collect and analyze data, and use probability to evaluate outcomes and make decisions. Students will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, radical, and rational functions. Students will use multiple representations, technology, applications and modeling in problem-solving contexts. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Exploring Data	**Designing Studies**
• Represent data visually and calculate statistical measures that describe the data set. (e.g., Construct and interpret a histogram for a student created data set.)	• Design a plan to collect data using an appropriate sampling method to solve a problem. (e.g., Design and conduct an experiment to determine the effect of a treatment.)
Functions and Modeling	**Probability and Informed Decisions**
• Explore expressions, functions, and models to highlight key features that provide insight into their structure, properties, and form. (e.g., Model situations using mathematics to provide an optimal solution.)	• Make inferences and justify conclusions from data and analyze decisions and strategies using probability concepts. (e.g., Compare experimental and theoretical probabilities to make informed decisions.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Applied Statistics:

Exploring Data	
Select appropriate graphical and numerical methods to explore data.	Standards 1-7
Designing Studies	
Design and implement a plan to collect and analyze data.	Standards 8-12
Functions and Modeling	
Explore expressions, functions, and models to describe numbers or relationships.	Standards 13-21
Probability and Informed Decisions	
Use probability to evaluate outcomes and make decisions.	Standards 22-29

Exploring Data

Cluster	Select appropriate graphical and numerical methods to explore data.	Can this student…?
M.ASHS.1	Generate appropriate ways to display various types of data. Instructional Note: Build on data displays introduced in prior courses.	• Organize, display, and interpret information in various graphs (e.g., scatter plots and line graphs) • Represent data with plots on the real number line (dot plots, histograms, and box plots) • Sketch a line of best fit for a set of data points • Determine the quartiles or interquartile range for a set of data
M.ASHS.2	Calculate appropriate measures of center, variability, and position for data. Instructional note: Include comparisons of mean vs. median, standard deviation vs. IQR.	• Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the context of the data sets accounting for

		possible effects of extreme data points (outliers) • Find the mean and standard deviation of a data set using technology • Choose a measure of central tendency based on the shape of the data distribution • Calculate median, mean, and interquartile range
M.ASHS.3	Use graphical displays and summary statistics to make conclusions. Informally develop the concept of statistical significance; a result that is unlikely to have occurred by chance alone. Instructional Note: Focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots) • Understand that statistics can be used to gain information about a population by examining a sample of the population
M.ASHS.4	Represent data in two variables to model relationships between quantities. Instructional Note: Students will use multiple representations with appropriate labels and scales.	• Organize, display, and interpret information in various graphs (e.g. scatter plots and line graphs) • Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Use relative frequencies calculated for rows or columns to describe possible associations between two variables • Compute (using technology) and interpret the correlation coefficient of a linear fit • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities

M.ASHS.5	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities. Instructional Note: Focus on form, strength, direction, and departures from a model based on data and context.	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association
M.ASHS.6	Compare characteristics of two data sets each represented in different ways (algebraically, graphically, numerically, and verbally). Instructional Note: Focus on applications and how key features relate to characteristics of a situation and select an appropriate model.	• Identify rate of change from a graph, table, equation, verbal representation, or in the context of a real-world situation • Identify a function that models a relationship between two quantities, and interpret key features of graphs (e.g., x-intercepts and y-intercepts) • Identify whether a function is increasing or decreasing over a given interval • Describe qualitatively the function relationship between two quantities • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)

M.ASHS.7	Use appropriate measures of center and spread to describe a distribution. Instructional Note: Emphasize that only some data are well described by a normal distribution.	• Choose a measure of central tendency based on the shape of the data distribution • Recognize the characteristics of a normal distribution • Describe data using or selecting the appropriate measure of central tendency • Find the mean and standard deviation of a set of data using technology • Calculate median, mean, and interquartile range • Identify possible outliers in a given set of data

Designing Studies

Cluster	**Design and implement a plan to collect and analyze data.**	**Can this student…?**
M.ASHS.8	Develop a process for making inferences about population parameters based on a random sample through data collection and analysis.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Represent data with plots on the real number line (dot plots, histograms, and box plots) • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences
M.ASHS.9	Evaluate the results from a given data-generating process to determine consistency between theoretical and experimental probabilities. Instructional	• Calculate experimental and theoretical probabilities • Approximate the probability of a chance event by collecting data on

	Note: Include the Law of Large Numbers.	the chance process that produces it and observing its long-run relative frequency, and then predict the approximate relative frequency given the probability • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.ASHS.10	Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain the importance of randomization in each method. Instructional Note: Emphasize that the way in which data is collected determines the scope and nature of the conclusions.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences
M.ASHS.11	Use data from a sample survey to estimate a population mean or proportion. Instructional Note: Develop the connection between sample size and margin of error.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots) • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Find the mean and standard deviation of a set of data using technology

M.ASHS.12	Design and conduct an experiment to compare two treatments. Instructional Note: Include randomization, replication, blocking, and control in the design.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots) • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Find the mean and standard of a set of data using technology • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)

Functions and Modeling

Cluster	**Explore expressions, functions, and models to describe numbers or relationships.**	**Can this student…?**
M.ASHS.13	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, simple rational and exponential functions.	• Solve various types of equations (e.g., linear, quadratic, square root, etc.) • Identify important quantities in a practical situation • Use substitution to verify algebraically the accuracy of the solution

M.ASHS.14	Develop the concept of a complex number i such that $i^2 = -1$. Understand that every complex number can be written in the form a ± bi with a and b real.	• Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation • Recognize when the quadratic formula gives complex solutions and write them as a $\pm$ bi for real numbers a and b
M.ASHS.15	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• Apply properties of operations to produce equivalent expressions with variables • Apply the commutative, associative, and distributive properties to algebraic expressions
M.ASHS.16	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.	• Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents • Know and apply the properties of exponents to generate equivalent numerical expressions
M.ASHS.17	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring • Factor a quadratic expression to reveal the zeros of the function it defines

M.ASHS.18	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.	• Relate rational number arithmetic to rational expression arithmetic • Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial
M.ASHS.19	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.	• Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number • Compute fluently with radical expressions
M.ASHS.20	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., solve $z = \frac{x-\mu}{\sigma}$ for σ and Margin of Error = $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ for n.) Instructional Note: While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.	• Solve multi-step equations • Solve simple equations with squared variables • Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution
M.ASHS.21	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include intercepts,	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions,

	intervals where the function is increasing, decreasing, positive, negative, relative extrema, symmetries, and end behavior. Instructional Note: Emphasize the selection of a model function based on the behavior of data in context.	identifying zeros when suitable factorizations are available, and showing end behavior • Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph

Probability and Informed Decisions

Cluster	**Use probability to evaluate outcomes and make decisions.**	**Can this student…?**
M.ASHS.22	Connect sampling variability and margin of error to generate and interpret plausible parameter values. Instructional Note: The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred by chance alone. Focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Develop a probability model and use it to find probabilities of events • Calculate and compare with rational numbers • Understand that statistics can be used to gain information about a population by examining a sample of the population • Calculate experimental and theoretical probabilities • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.ASHS.23	Interpret results from a randomized experiment comparing two treatments. Use	• Develop a probability model and use it to find probabilities of events

	simulations to decide if experimental results are significant. Instructional Note: Develop informally the comparison of an observed result and an established probability value (for example $p \leq 0.05$).	• Calculate and compare with rational numbers • Understand that statistics can be used to gain information about a population by examining a sample of the population • Understand that random sampling tends to produce representative samples and support valid inferences • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center, and spread in the content of the data sets, accounting for possible effects of extreme data points (outliers) • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.ASHS.24	Evaluate claims based on data reports. Instructional Note: Data reports can be gathered from media.	• Understand that statistics can be used to gain information about a population by examining a sample of the population

		• Understand that random sampling tends to produce representative samples and support valid inferences • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Calculate experimental and theoretical probabilities • Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association
M.ASHS.25	Use probability rules to make fair decisions. Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these concepts with long-run frequency • Analyze both costs and benefits in different contextual situations • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and then predict the approximate relative frequency given the probability • Calculate and compare with rational numbers • Find probabilities of compound

		events using organized lists, tables, tree diagrams, and simulations • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Define conditional probability and independence • Determine the probability of a simple event • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.ASHS.26	Use two-way tables, tree diagrams, Venn diagrams, or 10 x 10 grids to model probabilities.	• Determine the probability of a simple event • Compute with rational numbers • Find probabilities of events using organized lists, tables, tree diagrams, and simulations • Use proportional relationships to solve multi-step percent problems
M.ASHS.27	Justify a decision using probability rules (e.g., product testing, medical testing, weather forecasting, marketing, or sports coaching decisions). Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these ideas with long-run frequency • Analyze both costs and benefits in different contextual situations • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and then predict the approximate relative frequency given the probability • Calculate and compare with rational numbers

		• Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Define conditional probability and independence • Determine the probability of a simple event
M.ASHS.28	Perform appropriate calculations for given outcomes and decisions based on expected values for non-normal distributions. Instructional Note: Focus on uniform, discrete, continuous (geometric areas), or games of chance.	• Make decisions based on probability and connect these ideas with long-run frequency • Analyze both costs and benefits in different contextual situations • Calculate and compare with rational numbers • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.ASHS.29	Given data from a normal distribution, use the mean and standard deviation to estimate population percentages. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Recognize that there are data sets for which such a procedure is not appropriate. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities).	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Represent data with plots on the real number line (dot plots, histograms, and box plots) • Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Find the mean and standard deviation of a set of data using technology • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative

		frequency, and then predict the approximate relative frequency given the probability

Support for Mathematics Standards

Financial Algebra/Mathematics

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and Mathematical Habits of Mind. Students in this course will focus on financial applications designed to deepen and extend understanding of mathematics. Students in Financial Algebra/Mathematics will communicate effectively, using accurate mathematical language in a financial context. Students will interpret and analyze various functions, graphs and data in order to make responsible and wise financial decisions in the context of their personal lives regarding banking services, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed through opportunities for exploration and investigation of mathematical content and financial literacy topics:

Mathematical Language in a Financial Context	**The Algebra of Finance**
• Demonstrate reasoning skills in developing and explaining and justifying sound financial decisions. • Communicate effectively, using accurate mathematical language in a financial context.	• Apply algebraic skills and concepts to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.
Financial Modeling with Functions	**Modeling with Data**
• Interpret and analyze various functions, graphs, and data to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.	• Create, interpret, and evaluate financial models to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Financial Algebra/Mathematics:

Mathematical Language in a Financial Context	
Communicate reasoning and decisions.	Standards 1-3
Algebra/Mathematics of Finance	
Use algebraic reasoning and techniques.	Standards 4-11
Financial Modeling with Functions	
Construct, graph, use, and interpret functions.	Standards 12-27
Financial Modeling with Data	
Represent, summarize, and evaluate data.	Standards 28-37

Mathematical Language in a Financial Context

Cluster	**Communicate reasoning and decisions.**	**Can this student…?**
M.FAM.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical decision making. (e.g., Demonstrate reasoning skills in creating and presenting a budget of monthly expenses based on a career pathway income, and analyze the soundness of the mathematical reasoning of others; determine outlook for a chosen career pathway and use the average salary to determine if the desired cost of living can be met.)	• Solve real-world and mathematical problems by writing and solving equations • Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related • Organize, display, and interpret information in tables and graphs (frequency tables, pictographs, and line plots)
M.FAM.2	Communicate with and about mathematics in a financial context.	• Organize, display, and interpret information in tables and graphs (frequency tables, pictographs, and line plots) • Compare representations of data; including graphs, tables, equations; within the context of the data • Make inferences and justify conclusions from sample surveys, experiments, and observational studies

		• Solve real-world and mathematical problems by writing and solving equations
M.FAM.3	Communicate with and about mathematics in writing and orally, both independently and collaboratively, by preparing financial plans (e.g., Plan for an emergency savings fund that will last three to six months in the case of loss of income; determine the total percentage of income paid to taxes or the percentage of total salary that a benefits package represents).	• Solve real-world and mathematical problems by writing and solving equations • Create equations and inequalities in one variable and use them to solve problems • Identify important quantities in a practical situation • Use variables to represent quantities and write expressions when solving a real-world or mathematical problem • Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest • Solve number and word problems using percent proportion, percent equation, or ratios

Algebra/Mathematics of Finance

Cluster	**Use algebraic reasoning and techniques.**	**Can this student…?**
M.FAM.4	Interpret parts of an expression or equation, such as terms, factors, and coefficients, in a variety of financial models including those found in stock markets, automobile financing and in banking contexts.	• Identify the terms, factors, and coefficients of an expression • Write, read, and evaluate expressions in which letters stand for numbers • Identify important quantities in a practical situation
M.FAM.5	Create and solve linear equations and inequalities in one variable and use them to solve problems in financial applications that may include, but are not limited to stock markets, automobile ownership, business modeling, or employment. (e.g., Calculate wages by hourly rates or pay periods to make decisions about pay in a real world context.)	• Solve various types of linear equations • Identify important quantities in a practical situation • Use substitution to verify algebraically the accuracy of the solution • Interpret the rate of change in the context of a real-world situation • Use ratio and rate reasoning to solve real-world and mathematical

		problems
M.FAM.6	Create equations in two or more variables to represent relationships between quantities in a financial context; graph equations on coordinate axes with labels and scales. Financial contexts may include, but are not limited to stock markets, automobile ownership, business modeling employment, banking, consumer debt, and independent living decisions regarding taxes or planning for retirement. (e.g., Create a linear expense equation based on fixed and variable expenses and graph choosing an appropriate scale and origin for the graph.)	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of data, including graphs, tables, equations, and context
M.FAM.7	Represent constraints in financial applications by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. (e.g., Create a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental; find the percentage of total salary that a benefits package represents; calculate taxes owed based on a given income and tax table and determine total percentage of income paid to taxes; calculate the gross pay and net pay using the FICA percentage (7.65%), retirement contribution, and worker's compensation insurance	• Write equations, inequalities, and systems given a real-world situation • Create an equation, inequality, or system to model a situation with two variables • Solve multistep ratio and percent problems • Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest

	(employer match).)	
M.FAM.8	Rearrange formulas for financial applications to highlight a quantity of interest, using the same reasoning as in solving equations. Know difference between growth and decay functions. (e.g., Solve the literal equation for exponential depreciation to find a depreciation rate and the literal equation for continuous interest to find the interest rate; apply the formula for average daily balance, (average daily balance*APR*days in billing cycle)/365, using literal equations with varying APRs and billing cycles.)	• Solve multi-step linear equations • Solve simple equations with squared variables • Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another • Interpret rate of change in the context of a real-world situation • Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution
M.FAM.9	Solve systems of linear equations exactly and approximately (e.g., with graphs) in making financial decisions, focusing on pairs of linear equations in two variables. (e.g., Create and solve a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental.)	• Create an equation, inequality, or system to model a situation with two variables • Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, and equations) • Understand that the solution to a system is the intersection of the two functions • Use substitution to verify algebraically the accuracy of the solution
M.FAM.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials that model financial situations. (e.g., Combine the polynomials that model income and expense to create a profit model.)	• Add, subtract, and multiply linear and/ or quadratic polynomials • Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients • Apply the distributive property in algebraic expressions
M.FAM.11	Solve quadratic equations in one variable in a financial context that may include, but are not limited to	• Solve quadratic equations by inspection, taking square roots, completing the square, the

	business modeling or employment decisions. (e.g., Given a quadratic equation that models a profit function, determine the break-even points; apply braking distance/stopping distance formulas to solve problems related to driving and safety data.)	quadratic formula, and factoring • Graph quadratic functions and show intercepts, maxima, and minima • Factor a quadratic expression to reveal the zeros of a function it defines • Evaluate square roots of perfect square numbers

Financial Modeling with Functions

Cluster	**Construct, graph, use, and interpret functions.**	**Can this student…?**
M.FAM.12	Use functions to model financial situations. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (e.g., Develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; use linear and polynomial functions to evaluate and communicate quantities as required by Internal Revenue Service and Social Security Administration regulations and to determine when and why the models may be discontinuous.)	• Write and graph constant, linear, piecewise-defined, and polynomial functions • Understand that a function is a rule that assigns to each input exactly one output • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form

M.FAM.13	Use function notation in financial applications, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a financial context. (e.g., In making decisions regarding retirement income, apply the formula $A(t) = Pe^{rt}$ to determine future value.)	• Understand that a function is a rule that assigns to each input exactly one output • Interpret the equation $y = mx + b$ as defining a linear function • Write, read, and evaluate expressions
M.FAM.14	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Use this relationship in analyzing financial situations. (e.g., Compare the linear function modeling simple interest with the exponential function modeling compound interest.)	• Recognize that the slope (rate of change) of a line is the same between any two given points, meaning it is constant • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is linear or nonlinear) • Interpret the rate of change in the context of a real-world situation • Find a function to model a linear relationship between two quantities • Use the properties of exponents to interpret expressions for exponential functions
M.FAM.15	For a function that models a relationship between two quantities in financial contexts, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (e.g., Write, graph, and interpret the	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior

	revenue (quadratic) function in comparison to the expense (linear) function using key features of the functions; reason quantitatively to compare subsidized and unsubsidized loans, as well as other forms of financial aid available to college students; calculate mortgage payments, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage.)	• Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph
M.FAM.16	Interpret the parameters in a linear or exponential function in terms of a context. (e.g., Investigate and compare, using technology and regression, historical data to determine if automobile depreciation follows a linear or exponential model.)	• Describe qualitatively the functional relationship between two quantities by analyzing a graph • Interpret rate of change in the context of a real-world situation • Compute (using technology) and interpret the correlation coefficient of a linear fit • Identify important quantities in a practical situation
M.FAM.17	Construct linear and exponential functions modeling financial contexts, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (e.g., Utilize linear and exponential functions to compare simple with compound interest.)	• Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Recognize that the slope (rate of a change) of a line is the same between any two given points • Interpret rate of change in the context of a real-world situation • Analyze graphs, identify

		situations, or solve problems with varying rates of change
M.FAM.18	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.	• For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship • Organize, display, and interpret information in scatter plots • Sketch a line of best fit for a set of data points
M.FAM.19	Calculate and interpret the average rate of change of a function modeling a financial context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (e.g., Examine depreciation trends.)	• Determine the slope (rate of change) of a line from a graph, table, or equation • Interpret the meaning of slope in the context of a real-world problem • Evaluate algebraic expressions in number and word problems
M.FAM.20	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (e.g., Graph the linear, quadratic, or exponential curve that models the demand versus supply functions and find the equilibrium point with and without technology.)	• For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship • Factor a quadratic expression to reveal the zeros of the function it defines • Distinguish between situations that can be modeled with linear functions and with exponential functions • Graph a linear equation with or without technology using a

		variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify and interpret the intercepts of a linear relation in number and word problems
M.FAM.21	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Utilize linear and exponential functions to compare simple with compound interest; calculate and compare using both the loan payment formula and payment schedules in table format, the monthly cost of purchasing an automobile, and discuss the feasibility of that payment in relation to monthly budget; compare two functions showing interest accrued when paying the minimum monthly payment over time compared to paying a larger monthly payment, and identify and compare the average rate of change between given time periods.)	• Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear) • Identify a function that models a relationship between two quantities, and interpret key features of graphs (e.g., x- and y-intercepts) • Identify rate of change from a graph, table, equation, verbal representation, or in the context of a real-world situation • Identify whether a function is increasing or decreasing over a given interval • Construct a function to model a relationship between two quantities
M.FAM.22	Graph linear and quadratic functions and show intercepts, maxima, and minima. (e.g., In the model of a profit function, determine the break-even points, the maximum possible loss, and the maximum profit.)	• Graph a linear or quadratic equation with or without technology using a variety of representations • Factor a quadratic expression to reveal the zeros of a function it defines • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify and interpret the

		intercepts of a linear relation in number and word problems • Use substitution of input values to determine output values • Interpret the equation y=mx+b as defining a linear function
M.FAM.23	Write a function that describes a relationship between two quantities in a financial context. (e.g., Calculate the costs associated with purchasing a vehicle, including leasing, purchasing with cash, or with a loan.)	• For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply properties of operations (e.g., the distributive property) to generate equivalent expressions
M.FAM.24	Identify the effect on functions that model financial situations of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (e.g., Identify the impact of a change in a constraint in a function that models retirement planning, business income and	• Identify and interpret intercepts of the graph of a function • Graph functions with technology • Recognize horizontal and vertical shifts from an equation • Recognize horizontal and vertical compressions and stretches from an equation • Interpret the meaning of each part (intercepts, slope, etc.) of a function

	expenses, or employment benefits.)	
M.FAM.25	Graph square root, cube root, and piecewise-defined functions that model financial situations, including step functions and absolute value functions. (e.g., Develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; analyze graphs of functions that model profit.)	• Use substitution of input values to determine output values of square root, cube root and absolute value equations • Identify issues of domain, range, and usefulness when examining piecewise-defined functions • Graph a linear or quadratic equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Determine if the slope (rate of change) is positive or negative over a given interval • Identify a function that models a relationship between two quantities and interpret key features of graphs
M.FAM.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model financial situations, and translate between the two forms. (e.g., Create recursive and explicit models of sequences related to retirement planning; amortization schedules for a loan; comparing subsidized and unsubsidized loans, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate	• Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of integers • Perform multi-step operations with rational numbers in number and word problems • Find missing terms in sequences • Determine the rule for a given sequence • Describe visual and numerical patterns

	mortgage.)	
M.FAM.27	Apply exponential formulas to solve for future and present value of investments by hand or with graphing technology. (e.g., $PV=FV*(1/(1+r)^n)$ and $A(t) = Pe^{rt}$)	• Evaluate expressions and use formulas to solve number and word problems involving exponential functions • Solve multi-step equations

Financial Modeling with Data

Cluster	**Represent, summarize, and evaluate data.**	**Can this student…?**
M.FAM.28	Represent data with plots on the real number line (dot plots, histograms, and box plots). Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.	• Determine the quartiles or interquartile range for a set of data • Interpret information on a graph • Locate points on a number line
M.FAM.29	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Fit regression lines to scatterplots and make predictions based on lines of best fit. Find and interpret correlation coefficients of regression equations in financial situations. (e.g., Use scatter plots to show correlation between two funds, two stocks or even a stock and the general market or in business situations to forecast sales or to compare revenue to the number of units sold.)	• Organize, display, and interpret information in various graphs (e.g. scatter plots and line graphs) • Compute (using technology) and interpret the correlation coefficient of a linear fit • Sketch a line of best fit for a set of data points • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and

		intercept • Determine if data has a positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data • Graph single points on a coordinate plane
M.FAM.30	Create a data display modeling financial situations. This may include, but is not limited to modeling the different savings options for a given investment at local banking establishments; calculating and comparing the monthly cost of purchasing an automobile using both the loan payment formula and payment schedules in table format; creating an amortization schedule through the use of spreadsheet technology and the formula tool for a loan given principle, term, monthly payment, and interest rate; creating representations of pay schedules using a variety of modeling technologies, and making decisions in a financial context based on those representations.	• Organize, display, and interpret information in various graphs (e.g. scatter plots and line graphs) • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Represent data with plots on the real number line (dot plots, histograms, and box plots) • Use proportional relationships to solve multi-step ratio and percent problems
M.FAM.31	Summarize categorical data in various forms (e.g., two-way frequency tables, circle graphs, segmented bar charts). Interpret relative frequencies in the context of the data in making financial decisions.	• Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table • Construct and interpret a two-way table summarizing data on two categorical variables

		collected from the same subjects • Use relative frequencies calculated for rows or columns to describe possible association between the two variables • Calculate with percentages
M.FAM.32	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Data sets may address, but are not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association • Identify outliers and determine their effect on the mean, median, and range of a set of data
M.FAM.33	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (e.g., Use units appropriately as a way to understand multi-step problems in relationship to understanding credit card fees and finance charges; compute distance, rate and time to solve problems to analyze driving and safety data, using single and multiple unit conversion; use and	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Solve unit rate problems • Interpret units consistently in formulae

	compare researched reaction times and vehicle velocity, as well as accepted equations to solve problems with braking distances.)	
M.FAM.34	Use financial models from automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning to solve problems.	• Compare representations of data including graphs, tables, equations, and context • Read and interpret information from a graph or table • Display numerical data in plots on a number line, including dot plots, histograms and box plots • Organize, display, and interpret information in various graphs (e.g., scatter plots and line graphs) • Write equations, inequalities, and systems given a real-world situation • Evaluate expressions and use formulas to solve number and word problems involving exponential functions
M.FAM.35	Evaluate reports based on data. Data may address, but is not limited to, planning for retirement or stock markets.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)
M.FAM.36	Use probability and expected value to analyze financial situations. (e.g., Model and compare automobile insurance policies.)	• Develop a probability model and use it to find probabilities of events • Make decisions based on probability and connect these concepts with long-run frequency • Analyze both costs and benefits in different contextual situations • Approximate the probability of a chance event by

		collecting data and observing its long-run frequency, and predict the approximate relative frequency given the probability • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulation
M.FAM.37	Evaluate the impact of taxes on business ownership including property tax, sales tax, social security, retirement, and disability benefits. Evaluate the impact of taxes on personal finance decisions.	• Describe graphically, algebraically, and verbally real-world phenomena as functions • Evaluate reports based on data • Construct simple equations and inequalities to solve real-world problems by reasoning about the quantities • Solve multistep ratio and percent problems

Support for Mathematics Standards

High School Mathematics IV-Trigonometry/Pre-calculus

All West Virginia teachers are responsible for classroom instruction that integrates content standards and Mathematical Habits of Mind. Students in this course will generalize and abstract learning accumulated through previous courses as the final springboard to calculus. Students will take an extensive look at the relationships among complex numbers, vectors, and matrices. They will build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students will expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in previous courses. They will enhance their understanding of probability by considering probability distributions and have previous experiences with series augmented. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Building Relationships among Complex Numbers, Vectors, and Matrices	**Analysis and Synthesis of Functions**
• Represent abstract situations involving vectors symbolically.	• Write a function that describes a relationship between two quantities. (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)
Trigonometric and Inverse Trigonometric Functions of Real Numbers	**Derivations in Analytic Geometry**
• Make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle.	• Make sense of the derivations of the equations of an ellipse and a hyperbola.

• Prove trigonometric identities and apply them problem solving situations.	
Modeling with Probability	**Series and Informal Limits**
• Develop a probability distribution. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple- choice test where each question has four choices, and find the expected grade under various grading schemes.)	• Apply mathematical induction to prove summation formulas.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Building Relationships among Complex Numbers, Vectors, and Matrices	
Perform arithmetic operations with complex numbers.	Standard 1
Represent complex numbers and their operations on the complex plane.	Standards 2-4
Represent and model with vector quantities.	Standards 5-7
Perform operations on vectors.	Standards 8-9
Perform operations on matrices and use matrices in applications.	Standards 10-16
Solve systems of equations.	Standards 17-18
Analysis and Synthesis of Functions	
Analyze functions using different representations.	Standard 19
Build a function that models a relationship between two quantities.	Standard 20
Build new functions from existing functions.	Standards 21-22
Trigonometric and Inverse Trigonometric Functions of Real Numbers	
Extend the domain of trigonometric functions	Standards 23-24

using the unit circle.	
Model periodic phenomena with trigonometric functions.	Standards 25-27
Prove and apply trigonometric identities.	Standard 28
Apply transformations of function to trigonometric functions.	Standard 29
Derivations in Analytic Geometry	
Translate between the geometric description and the equation for a conic section.	Standard 30
Explain volume formulas and use them to solve problems.	Standard 31
Modeling with Probability	
Calculate expected values and use them to solve problems.	Standards 32-35
Use probability to evaluate outcomes of decisions.	Standard 36
Series and Informal Limits	
Use sigma notation to evaluate finite sums.	Standards 37-38
Extend the geometric series to infinite geometric series.	Standards 39-40

Building Relationships among Complex Numbers, Vectors, and Matrices

Cluster	**Perform arithmetic operations with complex numbers.**	**Can this student…?**
M.4HSTP.1	Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers. Instructional Note: In Math II students extended the number system to include complex numbers and performed the operations of addition, subtraction, and multiplication.	• Use $i^2 = -1$ and the commutative, associative, and distributive properties to multiply complex conjugates • Identify the real and imaginary part of a complex number • Use the identity property of multiplication • Multiply polynomials
Cluster	**Represent complex numbers and their operations on the complex plane.**	**Can this student…?**
M.4HSTP.2	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain	• Find the moduli (magnitude) of a complex number • Identify the real and imaginary part of a complex number

	why the rectangular and polar forms of a given complex number represent the same number.	• Find the arctangent of a number • Apply the definition of the trigonometric functions, sine, cosine, and tangent • Apply the Pythagorean Theorem • Graph points on a coordinate plane
M.4HSTP.3	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.	• Use graphical representation to subtract vectors • Use graphical representation to add vectors • Rewrite vector subtraction v – w as v + (–w) • Represent a point as a vector • Represent complex numbers on the complex plane in rectangular and polar form
M.4HSTP.4	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.	• Find the moduli (magnitude) of a complex number • Apply the distance and midpoint formula • Represent complex numbers in rectangular form
Cluster	**Represent and model with vector quantities.**	**Can this student…?**
M.4HSTP.5	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). Instructional Note: This is the student's first experience with vectors. The vectors must be represented both geometrically and in component form with emphasis on vocabulary and symbols.	• Apply the Pythagorean Theorem to find the distance between two points in a coordinate system • Graph points on a coordinate plane
M.4HSTP.6	Find the components of a vector by subtracting the coordinates of an	• Represent vector quantities as directed line segments

	initial point from the coordinates of a terminal point.	
M.4HSTP.7	Solve problems involving velocity and other quantities that can be represented by vectors.	• Find the arctangent of a number • Use properties of right triangles to solve problems • Identify opposite and adjacent sides in a triangle from a given angle
Cluster	**Perform operations on vectors.**	**Can this student…?**
M.4HSTP.8	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.	• Represent vector quantities as directed line segments
M.4HSTP.9	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). b. Compute the magnitude of a scalar multiple cv using $\|c\mathbf{v}\| = \|c\| \times \|\mathbf{v}\|$. Compute the direction of c**v** knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of cv is either along **v** (for c > 0) or against **v** (for c < 0).	• Apply distance formula • Represent vector quantities as directed line segments

Cluster	Perform operations on matrices and use matrices in applications.	Can this student…?
M.4HSTP.10	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network).	• Interpret expressions that represent a quantity in terms of its context • Identify important quantities in a practical situation
M.4HSTP.11	Multiply matrices by scalars to produce new matrices (e.g., as when all of the payoffs in a game are doubled).	• Use matrices to represent and manipulate data
M.4HSTP.12	Add, subtract and multiply matrices of appropriate dimensions.	• Use matrices to represent and manipulate data
M.4HSTP.13	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. Instructional Note: This is an opportunity to view the algebraic field properties in a more generic context, particularly noting that matrix multiplication is not commutative.	• Apply the commutative, associative, and distributive properties to algebraic expressions
M.4HSTP.14	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	• Apply the properties of operations to generate equivalent expressions • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply properties of identities for real numbers
M.4HSTP.15	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	• Use matrices to represent and manipulate data
M.4HSTP.16	Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. Instructional Note: Matrix multiplication of a 2 x 2 matrix by a vector can be interpreted as transforming points or regions in the plane to different points	• Add, subtract, and multiply matrices of appropriate dimensions • Multiply a vector by a matrix of suitable dimensions • Understand that a two-dimensional figure is congruent to another if the second can be obtained from the

	or regions. In particular a matrix whose determinant is 1 or -1 does not change the area of a region.	first by a sequence of rotations, reflections and translations
Cluster	**Solve systems of equations.**	**Can this student…?**
M.4HSTP.17	Represent a system of linear equations as a single matrix equation in a vector variable.	• Multiply a vector by a matrix of suitable dimensions to produce another vector • Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x) • Solve a system of equations
M.4HSTP.18	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). Instructional Note: Students have earlier solved two linear equations in two variables by algebraic methods.	• Add, subtract and multiply matrices of appropriate dimensions • Multiply a vector by a matrix of suitable dimensions • Apply properties of inverses for real numbers

Analysis and Synthesis of Functions

Cluster	**Analyze functions using different representations.**	**Can this student…?**
M.4HSTP.19	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Instructional Note: This is an extension of graphical analysis from Math III or Algebra II that develops the key features of graphs with the exception of asymptotes. Students examine vertical, horizontal, and oblique asymptotes by considering limits. Students should note the	• Use the zeros to construct a rough graph of the function • Graph piecewise-defined functions • Graph linear and quadratic functions and show intercepts, maxima, and minima • Highlight issues of domain, range and usefulness when examining piecewise-defined functions • Factor a quadratic expression to reveal the zeros of the function it defines • Identify and interpret the

	case when the numerator and denominator of a rational function share a common factor. Utilize an informal notion of limit to analyze asymptotes and continuity in rational functions. Although the notion of limit is developed informally, proper notation should be followed.	intercepts of a linear equation
Cluster	**Build a function that models a relationship between two quantities.**	**Can this student…?**
M.4HSTP.20	Write a function that describes a relationship between two quantities, including composition of functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	• Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function • Combine standard function types using arithmetic operations • Apply properties of operation as strategies to add, subtract, factor, and expand linear expressions with rational coefficients
Cluster	**Build new functions from existing functions.**	**Can this student…?**
M.4HSTP.21	Find inverse functions. Instructional Note: This is an extension of concepts from Math III where the idea of inverse functions was introduced. a. Verify by composition that one function is the inverse of another. b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Instructional Note: Students must realize that inverses created through function composition produce	• Write a function that describes a relationship between two quantities, including composition of functions • Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes • Highlight issues of domain, range and usefulness when examining piecewise-defined functions • Solve an equation of the form

	the same graph as reflection about the line y = x. c. Produce an invertible function from a non-invertible function by restricting the domain. Instructional Note: Systematic procedures must be developed for restricting domains of non-invertible functions so that their inverses exist.	f(x) = c for a simple function f that has an inverse and write an expression for the inverse • Relate the domain of a function to its graph • Solve rational, radical, and exponential equations
M.4HSTP.22	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	• Graph exponential and logarithmic functions, showing intercepts and end behavior • Evaluate logarithms using technology • Apply properties of logarithms • For exponential models, express as a logarithm the solution to a $b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e • Use rules of exponents to simplify numeric and algebraic expressions

Trigonometric and Inverse Trigonometric Functions of Real Numbers

Cluster	Extend the domain of trigonometric functions using the unit circle.	Can this student…?
M.4HSTP.23	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. Instructional Note: Students use the extension	• Explain how the unit circle enables the extension of trigonometric functions interpreted as radian measures of angles around the unit circle • Apply the radian measure of an angle as the length of the arc on the unit circle

	of the domain of the trigonometric functions developed in Math III to obtain additional special angles and more general properties of the trigonometric functions.	subtended by the angle • Use properties of circles to solve problems involving arcs formed by central angles • Use properties of right triangles to solve problems • Identify opposite and adjacent side in a right triangle from a given angle
M.4HSTP.24	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	• Explain how the unit circle enables the extension of trigonometric functions interpreted as radian measures of angles around the unit circle • Recognize even and odd functions from their graphs and algebraic expressions • Write a function to reveal the symmetry of the graph • Graph trigonometric functions showing the period • Graph functions expressed symbolically and show key features of the graph
Cluster	**Model periodic phenomena with trigonometric functions.**	**Can this student…?**
M.4HSTP.25	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	• Find inverse functions • Describe qualitatively the function relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing) • Graph functions expressed symbolically and show key features of the graph • Read and interpret information from a graph or table
M.4HSTP.26	Use inverse functions to solve	• Represent constraints by

	trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. Instructional Note: Students should draw analogies to the work with inverses in the previous unit.	equations or inequalities and interpret solutions as viable or non-viable options in modeling context • For a function that models a relationship between two quantities, interpret key features of the graphs in terms of quantities • Create equations in two or more variables to represent relationships between quantities • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Find inverse functions
M.4HSTP.27	Solve more general trigonometric equations. (e.g., $2\sin^2 x + \sin x - 1 = 0$ can be solved using factoring.)	• Represent constraints by equations or inequalities and interpret solutions as viable or non-viable options in modeling context • Solve an equation of the form f(x) = c for a simple function and write an expression for the inverse • Use the process of factoring and completing the square in a quadratic function to show zeros
Cluster	**Prove and apply trigonometric identities.**	**Can this student…?**
M.4HSTP.28	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	• Use the structure of an expression to identify ways to rewrite it • Explain how the unit circle enables the extension of trigonometric functions interpreted as radian measures of angles around the unit circle

		• Apply the radian measure of an angle as the length of the arc on the unit circle subtended by the angle • Use properties of circles to solve problems involving arcs formed by central angles • Use properties of right triangles to solve problems • Identify opposite and adjacent side in a triangle from a given angle
Cluster	**Apply transformations of function to trigonometric functions.**	**Can this student…?**
M.4HSTP.29	Graph trigonometric functions showing key features, including phase shift. Instructional Note: In Math III, students graphed trigonometric functions showing period, amplitude and vertical shifts.	• Graph trigonometric functions showing period, midline, and amplitude • Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), and f(x + k) for specific values of k • Identify a function that models a relationship between two quantities and interpret key features of graphs • Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

Derivations in Analytic Geometry

Cluster	**Translate between the geometric description and the equation for a conic section.**	**Can this student…?**
M.4HSTP.30	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Instructional Note: In Math II students derived the equations of circles and parabolas. These derivations provide meaning to the otherwise arbitrary constants in the formulas.	• Complete the square in a quadratic expression • Rewrite or simplify algebraic expressions • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system
Cluster	**Explain volume formulas and use them to solve problems.**	**Can this student…?**
M.4HSTP.31	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. Instructional Note: Students were introduced to Cavalieri's principle in Math II.	• Use dissection arguments and Cavalieri's principle to give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. • Calculate the volume of a cylinder, pyramid, cone, and sphere • Calculate the circumference and area of a circle

Modeling with Probability

Cluster	Calculate expected values and use them to solve problems.	Can this student…?
M.4HSTP.32	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. Instructional Note: Although students are building on their previous experience with probability in middle grades and in Math II and III, this is their first experience with expected value and probability distributions.	• Describe events as subsets of a sample space • Find probabilities of events using organized lists, table, tree diagrams, and simulation • Apply histograms representing values of the random variable and their respective probabilities on the real number line • Determine the probability of a simple event • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
Cluster	**Explain volume formulas and use them to solve problems.**	**Can this student…?**
M.4HSTP.33	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	• Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space • Describe events as subsets of a sample space • Find probabilities of events using organized lists, table, tree diagrams, and simulation • Apply histograms representing values of the random variable and their respective probabilities on the real number line • Determine the probability of a simple event • Understand that the probability

		of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
M.4HSTP.34	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)	• Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space • Describe events as subsets of a sample space • Find probabilities of events using organized lists, table, tree diagrams, and simulation • Apply histograms representing values of the random variable and their respective probabilities on the real number line • Determine the probability of a simple event • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
M.4HSTP.35	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? Instructional Note: It is important that students	• Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space • Describe events as subsets of a sample space • Find probabilities of events using organized lists, table, tree diagrams, and simulation • Apply histograms representing values of the random variable and their respective

	can interpret the probability of an outcome as the area under a region of a probability distribution graph.	probabilities on the real number line • Determine the probability of a simple event • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
Cluster	**Use probability to evaluate outcomes of decisions.**	**Can this student…?**
M.4HSTP.36	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. (e.g., Find the expected winnings from a state lottery ticket or a game at a fast food restaurant.) b. Evaluate and compare strategies on the basis of expected values. (e.g., Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.)	• Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space • Describe events as subsets of a sample space • Find probabilities of events using organized lists, table, tree diagrams, and simulation • Apply histograms representing values of the random variable and their respective probabilities on the real number line • Determine the probability of a simple event • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring

Series and Informal Limits

Cluster	Use sigma notations to evaluate finite sums.	Can this student…?
M.4HSTP.37	Develop sigma notation and use it to write series in equivalent form. For example, write $\sum_{i=1}^{n} (3i^2 + 7)$ as $3\sum_{i=1}^{n} i^2 + 7\sum_{i=1}^{n} 1$.	• Write arithmetic and geometric sequences with an explicit formula • Determine the rule for a given sequence • Describe visual and numerical patterns
M.4HSTP.38	Apply the method of mathematical induction to prove summation formulas. For example, verify that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Instructional Note: Some students may have encountered induction in Math III in proving the Binomial Expansion Theorem, but for many this is their first experience.	• Prove polynomial identities and use them to describe numerical relationships • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Factor higher order expressions (e.g., greatest common factor, grouping, difference of squares, difference of cubes, sum of cubes, etc.) • Reason from expressions with numerical coefficients to those with variable coefficients, understanding the properties of one apply to the other
Cluster	**Extend geometric series to infinite geometric series.**	**Can this student…?**
M.4HSTP.39	Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Instructional Note: In Math I, students described geometric sequences with explicit formulas. Finite geometric series were developed in Math III.	• Derive the formula for the sum of geometric series • Write geometric sequences with an explicit formula • Determine the rule for a given sequence • Describe visual and numerical patterns

M.4HSTP.40	Apply infinite geometric series models. For example, find the area bounded by a Koch curve. Instructional Note: Rely on the intuitive concept of limit developed in unit 2 to justify that a geometric series converges if and only if the ratio is between -1 and 1.	• Derive the formula for the sum of geometric series • Write geometric sequences with an explicit formula • Determine the rule for a given sequence • Describe visual and numerical patterns

Support for Mathematics Standards

Technical Transition Mathematics for Seniors

All West Virginia teachers are responsible for classroom instruction that integrates content standards and Mathematical Habits of Mind. Technical Transition Mathematics for Seniors will solidify quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use fundamental concepts of algebra, geometry, introductory trigonometry, and statistical analysis. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity	**Algebra: Seeing Structure in Expressions**
Develop an understanding of basic operations, equivalent representations, and properties of the real number system.	Create equations or inequalities that model physical situations.
Functions: Interpreting Functions	**Geometry/Trigonometry**
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent, and interpret functions and their applications.	• Solve application problems by calculating area or surface area in two-dimensional object or volume in three-dimensional objects. • Understand and apply the Pythagorean Theorem for solving real-world problems. (e.g. checking accuracy on gate construction, conduit bending).
Modeling	**Statistics - Interpreting Categorical & Quantitative Data**
• Create and use two- and three-dimensional representation of authentic situations in problem solving. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Analyze and interpret tables, charts and graphs. (e.g. interpret a body mass index (BMI) chart). • Distinguish between correlation and causation.

Finance Mathematics	
Determine, represent and analyze mathematical models for personal finance.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Technical Transition Mathematics for Seniors:

Number and Quantity	
Math as a language	Standards 1-3
Math and Measurement	Standards 4-6
The Real Number System	Standards 7-8
Algebra: Seeing Structure in Expressions	
Understand the connections between proportional relationship, lines, and linear equations.	Standards 9-10
Create equations that describe numbers or relationships.	Standards 11-14
Solve systems of equations.	Standards 15-16
Functions: Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 17
Analyze functions using different representations.	Standards 18-20
Build a function that models a relationship between two quantities.	Standards 21-22
Geometry/Trigonometry	
Visualize relationships between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 23-26
Use geometric theorems and formulas to solve problems.	Standards 27-31
Define trigonometric ratios and solve problems involving right triangles.	Standards 32-33
Modeling	
Concrete geometric representation (physical modeling).	Standards 34-35
Summarize, represent, and interpret data on	Standards 36-39

two quantitative variables.	
Statistics – Interpreting Categorical & Quantitative Data	
Conduct statistical analysis.	Standards 40-43
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 44-48
Finance Mathematics	
Understand financial models.	Standards 49-50
Personal use of finance.	Standards 51-52

Number and Quantity

Cluster	Math as a language	Can this student…?
M.TTMS.1	Demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.	• Solve real-world and mathematical problems by writing and solving equations • Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related • Identify when two expressions are equivalent • Apply the properties of operations to generate equivalent expressions
M.TTMS.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.	• Organize, display, and interpret information in tables and graphs (frequency tables, pictographs, and line plots) • Compare representations of data; including graphs, tables, equations; within the context of the data • Make inferences and justify conclusions from sample surveys, experiments, and observational studies • Solve real-world and mathematical problems by writing and solving equations • Create equations and inequalities in one variable and use them to solve problems

		• Identify important quantities in a practical situation • Use substitution to verify algebraically the accuracy of the solution • Use variables to represent quantities and write expressions when solving a real-world or mathematical problem
M.TTMS.3	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Solve unit rate problems • Interpret units consistently in formulae
Cluster	**Math and Measurement**	**Can this student…?**
M.TTMS.4	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• Round numbers to appropriate levels of accuracy
M.TTMS.5	Solve real-life problems requiring conversion of units using dimensional analysis for measurements in English and metric systems. Solve problems involving multiple units of measurement. (e.g. converting between currencies, calculating dosages of medicine, trip planning from miles to kilometers.)	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Manipulate and transform units when multiplying or dividing quantities • Solve unit rate problems
M.TTMS.6	Distinguish between proportional and non-proportional situations, apply proportional reasoning when appropriate, solve for an unknown quantity in proportional situations; apply scale factors to perform indirect measurements. (e.g., maps, blueprints, concentrations, dosages, and densities)	• Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal description, identify the unit rate (constant of proportionality) • Recognize and represent proportional relationships between quantities • Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane • Solve linear equations • Identify corresponding parts of similar and congruent figures • Represent proportional relationships in multiple ways (table of values,

		ordered pairs, graph, unit rate, and constant of proportionality) • Calculate and explain unit rate • Use equations to represent proportional relationships between quantities • Use proportional relationships to solve multi-step problems
Cluster	**The Real Number System**	**Can this student…?**
M.TTMS.7	Perform operations using positive and negative numbers, fractions, absolute value, decimals, percentages, and scientific notation. (e.g., given the cost of a project, determine what percentage of the budget went for salaries, percent of increase/ decrease)	• Know and apply properties of integer exponents • Apply properties of operations as strategies to multiply and divide rational numbers • Apply properties of operations as strategies to add and subtract rational numbers • Apply previous understandings of multiplication and division and of fractions to multiply and divide rational numbers • Apply previous understandings of addition and subtraction to add and subtract rational numbers • Understand the absolute value of a rational number as its distance from zero on the number line
M.TTMS.8	Solve real-world problems in a variety of contexts by representing quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as scientific notation and the meaning of a fraction or percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages.	• Solve real-world and mathematical problems involving the four operations • Use proportional relationships to solve multi-step ratio and percent problems • Perform multi-step operations with rational numbers in number and word problems • Convert a rational number to a decimal

Algebra – Seeing Structure in Expressions

Cluster	Understand the connections between proportional relationship, lines, and linear equations.	Can this student…?
M.TTMS.9	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (e.g., Labor cost per time, material cost per job)	• Represent proportional relationships in multiple ways (table of values, ordered pairs, graph, unit rate, and constant of proportionality) • Plot points in a coordinate plane • Calculate and explain unit rate
M.TTMS.10	Solve application problems using direct and inverse variation equations. (e.g., determine the mechanical advantage of gears, Ohm's Law)	• Compare properties of two functions each represented in a different way • Recall multiple representations of proportional relationships (table of values, ordered pairs, graph, unit rate, and constant of proportionality) • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form • Given a list of ordered pairs in a table or graph, identify either verbally or algebraically the rule used to generate and record the results • Identify rate of change/slope when given a linear function • Recall $y = mx + b$ and how to determine values for each variable using tables, graphs, or verbal descriptions
Cluster	**Create equations that describe numbers or relationships.**	**Can this student…?**

M.TTMS.11	Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem.	• Write a function that describes a relationship between two quantities • Solve systems of linear equations, including cases with one solution, no solution, and infinitely many solutions • Solve linear equations in one variable, including cases with one solution, no solution, and infinitely many solutions • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Use variables to represent two quantities in a real-world problem that change in relationship to one another • Translate between models or verbal phrases and algebraic expressions
M.TTMS.12	Analyze real-world problem situations and use variables to construct and solve equations and inequalities in one variable. Include equations arising from linear functions and simple rational and exponential functions. (e.g., using spreadsheet functions, determine sale price of items)	• Solve various types of equations (e.g., linear, square root, etc.) • Construct simple equations and inequalities to solve problems by reasoning about the quantities • Identify important quantities in a practical situation • Solve one-step equations and inequalities • Use substitution to verify algebraically the accuracy of the solution • Evaluate expressions using exponents • Write, read, and evaluate expressions
M.TTMS.13	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (e.g., profit vs. number of units, cost vs. number of units, resistance vs. current)	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation

		• Compare representations of data, including graphs, tables, equations, and context
M.TTMS.14	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R)	• Solve simple equations with squared variables • Explain each step in solving a simple equation as following from the properties of equality • Solve multi-step equations
Cluster	**Solve systems of equations.**	**Can this student…?**
M.TTMS.15	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (e.g., child care facility – sq. footage to number of children; solving electrical current in a circuit with multiple paths, Break-Even point)	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equation) • Understand that the solution to a system is the intersection of the two functions • Use substitution to verify algebraically the accuracy of the solution
M.TTMS.16	Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).	• Graph absolute value, quadratic, and higher order equations with or without technology • Solve a linear-quadratic system of equations • Solve a system of linear equations • Understand that the solution to a system is the intersection of the two functions • Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations)

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**	**Can this student…?**
M.TTMS.17	Understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	• Understand that a function is a rule that assigns to each input exactly one output • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form
Cluster	**Analyze functions using different representations.**	**Can this student…?**
M.TTMS.18	Interpret the parameters in a linear function in terms of a context.	• Describe qualitatively the functional relationship between two quantities by analyzing a graph • Interpret rate of change in the context of a real-world situation
M.TTMS.19	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.	• Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form • Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions • Differentiate between linear and non- linear in graphical representations
M.TTMS.20	Describe qualitatively the functional relationship between two quantities by analyzing a graph.	• Construct or complete a table of values to solve problems associated with a given relationship • Organize, display, and interpret information in line graphs

Cluster	**Build a function that models a relationship between two quantities.**	**Can this student…?**
M.TTMS.21	Represent application problems as linear equations. Write a function that describes a relationship between two quantities. (e.g., level of education versus pay; rate of speed versus fuel consumption; caloric intake versus expenditure)	• Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Apply the properties of operations (e.g., the distributive property) to generate equivalent expressions • Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Interpret the equation y=mx + b as defining a linear function
M.TTMS.22	Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Use substitution to verify algebraically the accuracy of the solution • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is linear or nonlinear) • Interpret rate of change in the context of a real-world situation • Recognize that the slope (rate of change) of a line is the same between any two given points, meaning it is constant

Geometry/Trigonometry

Cluster	Visualize relationships between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Can this student…?
M.TTMS.23	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids
M.TTMS.24	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	• Calculate volume and surface area of shapes • Identify basic three-dimensional shapes • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units
M.TTMS.25	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot, airflow per cubic foot).	• Calculate area and volume of shapes • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units
M.TTMS.26	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).	• Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

Cluster	Use geometric theorems and formulas to solve problems.	Can this student…?
M.TTMS.27	Use theorems about triangles to solve real-world application problems. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Recognize the attributes of various types of triangles • Notice the conditions which determine a unique triangle, more than one triangle, or no triangle
M.TTMS.28	Understand and apply the Pythagorean Theorem for solving real-world problems. (e.g., checking accuracy on gate construction, conduit bending)	• Use geometric models and equations to investigate the meaning of the square of a number and the relationship to its positive square root • Identify and classify triangles according to the measures of the interior angles and the lengths of the sides; relate triangles based upon their hierarchical attributes • Write and solve algebraic equations • Utilize strategies to solve real-world problems • Use coordinates and absolute value to find distances between points
M.TTMS.29	Solve application problems by calculating area and surface area for two-dimensional objects. (e.g., calculate the cost of heating a building based on square footage)	• Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures • Find the area of circles and various polygons • Recognize the two-dimensional elements of three-dimensional figures • Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., face, edge, vertex, and base) to identify and compare properties of solid figures

		• Apply the formula for the area of rectangles • Recall that the whole is equal to the sum of its parts • Evaluate algebraic expressions
M.TTMS.30	Solve application problems by calculating volume for three-dimensional objects using formulas for cylinders, pyramids, prisms, cones, and spheres. (e.g., compute amount of cement needed for a sidewalk, amount of water in a fire hose, amount of air in ductwork)	• Calculate the volume of a cylinder, pyramid, cone, and sphere • Use models to find volume for prisms and cylinders as the product of the area of the base and the height • Calculate the volume of prisms in number and word problems • Write and solve equations from models and word problems including units • Apply the formulas for volume of right rectangular prisms • Find the volume of a right rectangular prism • Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures • Find the area of circles and various polygons
M.TTMS.31	Solve application problems by calculating circumference, area, radius, diameter, area of sector, arc length of a circle with appropriate unit labels. (e.g., develop a circular watering system)	• Calculate the circumference and area of a circle • Use proportional relationships to solve multi-step ratio problems • Know and apply the formulas for the area and circumference of a circle
Cluster	**Define trigonometric ratios and solve problems involving right triangles.**	**Can this student…?**
M.TTMS.32	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (e.g., angles of depression/elevation, conduit bending)	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • In a right triangle, identify opposite and adjacent sides to an angle

		• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure
M.TTMS.33	Identify and apply special right triangle relationships, 30-60-90 and 45-45-90. (e.g., isometric drawing in drafting, conduit bending)	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Identify properties of right triangles • Rewrite expressions involving radicals and rational exponents using the properties of exponents

Modeling

Cluster	**Concrete geometric representation (physical modeling).**	**Can this student…?**
M.TTMS.34	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.	• Apply geometric methods to solve design problems • Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation • Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Describe the two-dimensional figures that result from slicing a three-dimensional figure

		• Represent three-dimensional figures using nets made up of rectangles and triangles
M.TTMS.35	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.	• Recognize the purposes and differences among sample surveys, experiments, and observational studies • Represent data with plots on the real number line (dot plots, histograms, and box plots) • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • Organize, display, and interpret information in various graphs (e.g., scatter plots and line graphs) • Explain how to draw inferences from data distributions • Recall how to summarize numerical data sets in relation to their context • Identify scatter plot, outlier, linear, quantitative, variable
Cluster	**Summarize, represent, and interpret data on two quantitative variables.**	**Can this student…?**
M.TTMS.36	Collect numerical bivariate data; represent data on two quantitative variables on a scatter plot; determine whether or not a relationship exists; if so, describe how the variables are related and select a function to model the data, justify the selection and use the model to make predictions.	• Organize, display, and interpret information in various graphs • Sketch a line of best fit for a set of data points • Determine if data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data
M.TTMS.37	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter	• Organize, display, and interpret information in line graphs

	plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	• Sketch a line of best fit for a set of data points • Determine if the data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data • Explain how to draw inferences from data distributions • Recall how to summarize numerical data sets in relation to their context
M.TTMS.38	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.	• Calculate slope (rate of change) from a variety of representations • Interpret the equation y = mx + b as defining a linear function • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities
M.TTMS.39	Compute (using technology) and interpret the correlation coefficient of a linear fit. Identify positive and negative correlations. (e.g., vehicle depreciation)	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept • Determine if data has positive association, negative association, or no association

Statistics - Interpreting Categorical & Quantitative Data

Cluster	Conduct statistical analysis.	Can this student...?
M.TTMS.40	Identify the population of interest, select an appropriate sampling technique and collect data.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
M.TTMS.41	Identify the variables to be used in a study.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences • Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape

M.TTMS.42	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association • Understand that random sampling tends to produce representative samples and support valid inferences • Understand that statistics can be used to gain information about a population by examining a sample of the population • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
M.TTMS.43	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.	• Recognize that a measure of center for a numerical data set summarizes all of its values with a single number • Identify mean, median, mode, and range and explain how to calculate each • Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions • Recognize that a statistical question is one that will require gathering data that has variability • Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.	Can this student…?
M.TTMS.44	Represent data with plots on the real number line (dot plots, histograms, and box plots).	• Determine the quartiles or interquartile range for a set of data • Interpret information on a graph • Locate points on a number line
M.TTMS.45	Analyze and interpret tables, charts and graphs. (e.g., interpret a body mass index (BMI) chart)	• Organize, display, and interpret information in tables and graphs (frequency, pictographs, and line plots) • Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects
M.TTMS.46	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Calculate median, mean, and interquartile range
M.TTMS.47	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association

M.TTMS.48	Distinguish between correlation and causation.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities

Finance Mathematics

Cluster	Understanding financial models.	Can this student…?
M.TTMS.49	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Business Loans, Auto, Mortgage, and/or Credit Card).	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of data including graphs, tables, equations, and context • Write a function that describes a relationship between two quantities • Identify rate of change from a graph, table, equation, or verbal representation • Describe qualitatively the functional relationship between two quantities by analyzing the graph (e.g., increasing or decreasing, linear or nonlinear, etc.)
M.TTMS.50	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit)	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Write a function that describes a relationship between two quantities • Compare representations of data including graphs, tables, equations, and context • Describe qualitatively the functional relationship between two quantities by analyzing the graph (e.g.,

		increasing or decreasing, linear or nonlinear, etc.) • Identify rate of change from a graph, table, equation, or verbal representation
Cluster	**Personal use of finance**	**Can this student…?**
M.TTMS.51	Research, develop and analyze personal budgets based on given parameters (e.g., Fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement and/or investment planning, etc.).	• Solve real-world and mathematical problems by writing and solving equations • Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies • Understand that positive and negative numbers are used together to describe quantities having opposite directions or values
M.TTMS.52	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.	• Solve real-world and mathematical problems by writing and solving equations • Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies • Understand that positive and negative numbers are used together to describe quantities having opposite directions or values

Support for Mathematics Standards

Transition Mathematics for Seniors

All West Virginia teachers are responsible for classroom instruction that integrates content standards and Mathematical Habits of Mind. Transition Mathematics for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: **The Real Number System** **The Complex Number System**	**Algebra:** **Seeing Structure in Expressions** **Arithmetic with Polynomials and Rational Expressions** **Creating Equations** **Reasoning with Equations and Inequalities**
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions and Building Functions	**Geometry: Geometric Measuring and Dimension, Expressing Geometric Properties with Equations, and Modeling with Geometry**
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates to prove geometric properties algebraically.
Statistics and Probability: Interpreting	

Categorical and Quantitative Data and Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Number and Quantity – The Real Number System	
Extend the properties of exponents to rational exponents.	Standards 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra - Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standards 5-6
Understand the connections between proportional relationship, lines, and linear equations.	Standards 7-9
Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 10
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 11-14
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 15
Solve equations and inequalities in one variable.	Standards 16-18
Solve systems of equations.	Standards 19-21
Represent and solve equations and inequalities graphically.	Standards 22-23

Functions: Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 24
Interpret functions that arise in applications in terms of the context.	Standards 25-28
Analyze functions using different representations.	Standards 29-35
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 36-37
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 38-39
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 40-41
Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 42
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 43-46
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 47-51
Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 52

Number and Quantity – The Real Number System

Cluster	Extend the properties of exponents to rational exponents.	Can this student…?
M.TMS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Solve unit rate problems • Interpret units consistently in formulae
M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• Round numbers to appropriate levels of accuracy

Number and Quantity – The Complex Number System

Cluster	Use complex numbers in polynomial identities and equations.	Can this student…?
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.	• Solve quadratic equations with real coefficients that have real solutions • Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b

Algebra – Seeing Structure in Expressions

Cluster	Interpret the structure of expressions.	Can this student…?
M.TMS.4	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	• Know and apply the properties of integer exponents to generate equivalent numerical expressions • Find factors, common factors, and the greatest common factors of expressions

Cluster	**Write expressions in equivalent forms to solve problems.**	**Can this student…?**
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	• Understand what different forms of a quadratic expression reveal • Evaluate algebraic expressions in number and word problems
M.TMS.6	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	• Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of integers • Find the common ratio of a geometric sequence • Find missing terms in sequences • Determine the rule for a given sequence • Describe visual and numerical patterns • Substitute values into a function
Cluster	**Understand the connections between proportional relationship, lines, and linear equations.**	**Can this student…?**
M.TMS.7	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	• Represent proportional relationships in multiple ways (table of values, ordered pairs, graph, unit rate, and constant of proportionality) • Plot points in a coordinate plane • Calculate and explain unit rate

M.TMS.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	• Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal description, identify the unit rate (constant of proportionality) • Recognize and represent proportional relationships between quantities • Use a coordinate plane to solve number and word problems • Describe the path between given points on the plane • Solve linear equations • Identify corresponding parts of similar and congruent figures
M.TMS.9	Solve linear equations in one variable.	• Use variables to represent quantities and construct simple equations to solve problems • Identify from a set of numbers which values satisfy a given equation or inequality • Apply properties of operation to add, subtract, factor, and expand linear expressions with rational coefficients • Simplify expressions and combine like terms

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	Perform arithmetic operations on polynomials.	Can this student…?
M.TMS.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.	• Add, subtract, and multiply linear and/or quadratic polynomials • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property in algebraic expressions

Algebra – Creating Equations

Cluster	Create equations that describe numbers or relationships.	Can this student…?
M.TMS.11	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.	• Solve various types of equations (e.g., linear, quadratic, square root, etc.) • Construct simple equations and inequalities to solve problems by reasoning about the quantities • Identify important quantities in a practical situation • Solve one-step equations and inequalities • Use substitution to verify algebraically the accuracy of the solution • Evaluate expressions using exponents • Write, read, and evaluate expressions
M.TMS.12	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Compare representations of

		data, including graphs, tables, equations and context
M.TMS.13	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	• Write equations, inequalities, and systems given a real-world situation
M.TMS.14	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	• Solve simple equations with squared variables • Explain each step in solving a simple equation as following from the properties of equality • Solve multi-step equations

Algebra – Reasoning with Equations and Inequalities

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**	**Can this student…?**
M.TMS.15	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.	• Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number • Compute fluently with radical expressions
Cluster	**Solve equations and inequalities in one variable.**	**Can this student…?**
M.TMS.16	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	• Use substitution to algebraically verify the accuracy of the solution • Solve one-step equations and inequalities
M.TMS.17	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable	• Use the distributive property and the order of operations to create equivalent expressions • Identify when two expressions are equivalent (e.g., 3y and y+y+y)

	argument to justify a solution method.	
M.TMS.18	Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	• Graph quadratic functions and show intercepts, maxima, and minima • Factor a quadratic expression to reveal the zeros of a function it defines • Evaluate square roots of perfect square numbers • Compute with radical expressions
Cluster	**Solve systems of equations.**	**Can this student…?**
M.TMS.19	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	• Graph a system of equations with or without technology • Apply the distributive property to an expression
M.TMS.20	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	• Understand that the solution to a system of two linear equations in two variables corresponds to the point of intersection of their graphs, because points of intersection satisfy both equations simultaneously • Solve a system of linear equations • Use substitution to verify algebraically the accuracy of the solution
M.TMS.21	Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solution of the equation $f(x) = g(x)$; find the solution approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).	• Graph absolute value, quadratic, and higher order equations with or without technology • Solve a linear-quadratic system of equations • Solve a system of linear equations • Understand that the solution to a system is the intersection of the two functions

		• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations)
Cluster	**Represent and solve equations and inequalities graphically.**	**Can this student…?**
M.TMS.22	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Understand that the solution to a system is the intersection of the two functions • Use substitution to verify algebraically the accuracy of the solution
M.TMS.23	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	• Graph single-variable inequalities on a number line using appropriate shading • Test points in the inequality to verify whether the point is a solution to the inequality • Understand the application of inequality symbols as they pertain to graphing

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**	**Can this student…?**
M.TMS.24	Understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	• Understand that a function is a rule that assigns to each input exactly one output • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form
Cluster	**Interpret functions that arise in applications in terms of the context.**	**Can this student…?**

M.TMS.25	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	• Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of integers • Perform multi-step operations with rational numbers in number and word problems • Find missing terms in sequences • Determine the rule for a given sequence • Describe visual and numerical patterns
M.TMS.26	Interpret the parameters in a linear or exponential function in terms of a context.	• Describe qualitatively the functional relationship between two quantities by analyzing a graph • Interpret rate of change in the context of a real-world situation
M.TMS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior • Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph
M.TMS.28	Distinguish between situations that can be modeled with linear functions and with exponential functions.	• Recognize that the slope (rate of change) of a line is the same between any two given points, meaning it is constant • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where

		the function is linear or nonlinear) • Interpret rate of change in the context of a real-world situation • Analyze graphs, identify situations, or solve problems with varying rates of change • Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs
Cluster	**Analyze functions using different representations.**	**Can this student…?**
M.TMS.29	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.	• Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form • Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions • Differentiate between linear and non-linear in graphical representations
M.TMS.30	Describe qualitatively the functional relationship between two quantities by analyzing a graph.	• Construct or complete a table of values to solve problems associated with a given relationship • Organize, display, and interpret information in line graphs
M.TMS.31	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.	• Identify and interpret intercepts of the graph of a function • Graph functions with technology • Recognize horizontal and vertical shifts from an equation

		• Recognize horizontal and vertical compressions and stretches from an equation • Construct a function to model a linear relationship between two quantities • Interpret the meaning of each part (intercepts, slope, etc.) of a function
M.TMS.32	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	• Graph quadratic functions • Factor a quadratic expression to reveal the zeros of the function it defines • Graph a linear equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify and interpret the intercepts of a linear relation in number and word problems • Interpret the equation y=mx+b as defining a linear function
M.TMS.33	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.	• Read and interpret information from a graph or table
M.TMS.34	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	• Understand that factoring is the reversal of the distributive property • Factor the greatest common factor from an expression • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Recognize situations in which a quantity grows of decays by a constant percent rate per unit interval relative to another

		• Evaluate algebraic expressions in number and word problems • Apply properties of operation as strategies to add, subtract, factor, and expand linear expressions with rational coefficients
M.TMS.35	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	• Identify rate of change from a graph, table, equation, verbal representation, or in the context of a real-world situation • Identify a function that models a relationship between two quantities and interpret key features of graphs (e.g., x- and y-intercepts) • Identify whether a function is increasing or decreasing over a given interval • Describe qualitatively the function relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing) • Construct a function to model a relationship between two quantities

Functions - Building Functions

Cluster	**Build a function that models a relationship between two quantities.**	**Can this student…?**
M.TMS.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	• Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation • Recognize that the slope (rate of a change) of a line is the same between any two given points • Interpret rate of change in the context of a real-world situation • Analyze graphs, identify situations, or solve problems with varying rates of

		change
M.TMS.37	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	• Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Rewrite or simplify algebraic expressions including the use of commutative, associative, and distributive properties, inverses and identities in number and word problems • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply properties of operations (e.g., the distributive property) to generate equivalent expressions

Geometry – Geometric Measuring and Dimension

Cluster	**Explain volume formulas and use them to solve problems.**	**Can this student…?**
M.TMS.38	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	• Calculate the circumference and area of a circle • Calculate the volume of a cylinder, pyramid, and cone
M.TMS.39	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	• Calculate the volume of a sphere, cylinder, pyramid, and cone • Calculate the circumference and area of a circle

Geometry – Expressing Geometric Properties with Equations

Cluster	Use coordinates to prove simple geometric theorems algebraically.	Can this student…?
M.TMS.40	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	• Apply the Pythagorean Theorem to right triangles • Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane • Prove the slope criteria for parallel and perpendicular lines
M.TMS.41	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).	• Apply the Pythagorean Theorem to find the distance between two points in the coordinate plane • Understand and apply concepts of perimeter and area in two-dimensional shapes

Geometry – Modeling with Geometry

Cluster	Apply geometric concepts in modeling situations.	Can this student…?
M.TMS.42	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).	• Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.	Can this student...?
M.TMS.43	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret linear models.	• Organize, display, and interpret information in various graphs • Sketch a line of best fit for a set of data points • Determine if data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data
M.TMS.44	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Calculate slope (rate of change) from a variety of representations • Interpret the equation y=mx+b as defining a linear function • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities
M.TMS.45	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	• Organize, display, and interpret information in line graphs • Sketch a line of best fit for a set of data points • Determine if the data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data • Explain how to draw inferences from data distributions • Recall how to summarize numerical data sets in relation to their context
M.TMS.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional	• Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way

	relative frequencies). Recognize possible associations and trends in the data.	table • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Use relative frequencies calculated for rows or columns to describe possible associations between two variables • Calculate with percentages
Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**	**Can this student…?**
M.TMS.47	Represent data with plots on the real number line (dot plots, histograms, and box plots).	• Determine the quartiles or interquartile range for a set of data • Interpret information on a graph
M.TMS.48	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Calculate median, mean, and interquartile range
M.TMS.49	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association
M.TMS.50	Compute (using technology) and interpret the correlation coefficient of a linear fit.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data,

		interpreting the slope and intercept • Determine if data has positive association, negative association, or no association
M.TMS.51	Distinguish between correlation and causation.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities

Statistics and Probability – Making Inferences and Justifying Conclusions

Cluster	**Understand and evaluate random processes underlying statistical experiments.**	**Can this student…?**
M.TMS.52	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences

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