High School Algebra I

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstractly and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – High School Algebra I

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will focus on four critical domains that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Expressions and Equations	Functions
• Interpret algebraic expressions and transforming them purposefully to solve problems (e.g., in solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P).	• Understand contextual relationships of variables and constants (e.g., Annie is picking apples with her sister; the number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples; what do the numbers 22 and 12 tell you about Annie's apple picking?).
Geometry	**Statistics and Probability**
• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances (e.g., find the area and perimeter of a real-world shape using a coordinate grid and Google Earth).	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model (e.g., use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Algebra I:

Expressions and Equations	
Interpret the structure of expressions and equations in terms of the context they model.	Standards 1-2
Extend the properties of exponents to rational exponents.	Standards 3-4
Write expressions in equivalent forms to solve problems.	Standard 5
Perform arithmetic operations on polynomials.	Standard 6
Create equations that describe numbers or relationships.	Standards 7-9
Solve equations and inequalities in one variable.	Standards 10-11
Solve systems of equations.	Standards 12-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Functions	
Understand the concept of a function and use function notation.	Standards 19-21
Interpret functions that arise in applications in terms of a context.	Standard 22
Analyze functions using different representations.	Standards 23-25
Build a function that models a relationship between two quantities.	Standards 26-27

Build new functions from existing functions.	Standard 28
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 29
Geometry	
Use coordinates to prove simple geometric theorems algebraically.	Standards 30-31
Statistics and Probability	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 32-34
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standard 35
Interpret linear models.	Standards 36-37

Expressions and Equations

Cluster	**Interpret the structure of expressions and equations in terms of the context they model.**
M.A1HS.1	Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. c. Interpret the parameters in a linear function or exponential function of the form $f(x) = a*b^x$ in terms of a context.
M.A1HS.2	Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Cluster	**Extend the properties of exponents to rational exponents.**
M.A1HS.3	Explain the connections between expressions with rational exponents and expressions with radicals using properties of exponents. Extend from application of properties of exponents for expressions with integer exponents.
M.A1HS.4	Rewrite expressions involving radicals, including simplifying, and rational exponents using the properties of exponents.

Cluster	Write expressions in equivalent forms to solve problems.
M.A1HS.5	Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression, when a=1 only, to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})12^t \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	Perform arithmetic operations on polynomials.
M.A1HS.6	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on linear or quadratic terms.

Cluster	Create equations that describe numbers or relationships.
M.A1HS.7	Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs.
M.A1HS.8	Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs.
M.A1HS.9	Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Cluster	Solve equations and inequalities in one variable.
M.A1HS.10	Solve linear equations including equations with coefficients represented by letters, simple exponential equations that rely on application of the laws of exponents, and compound linear inequalities in one variable.
M.A1HS.11	Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square when a=1 only, and the quadratic formula, as appropriate for the initial form of the equation. a. Recognize the concept of complex solutions when the quadratic formula gives complex solutions. b. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$. Derive the quadratic formula from this method of completing the square.

Cluster	**Solve systems of equations.**
M.A1HS.12	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve simple cases by inspection (e.g., $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6). c. Solve real-word and mathematical problems leading to two linear equations in two variables (e.g., given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair).
M.A1HS.13	Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set.
M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.A1HS.15	Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically.

Cluster	**Represent and solve equations and inequalities graphically.**
M.A1HS.16	Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.
M.A1HS.17	Explain why the x-coordinates of the points where the graphs of the linear and/or exponential equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).
M.A1HS.18	Graph the solutions of a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Cluster	**Understand the concept of a function and use function notation.**
M.A1HS.19	Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.
M.A1HS.20	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.
M.A1HS.21	Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n)+ f(n-1)$ for $n \geq 1$).

Cluster	Interpret functions that arise in applications in terms of a context.
M.A1HS.22	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes. a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative. b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior.

Cluster	Analyze functions using different representations.
M.A1HS.23	Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph. a. For linear functions, focus on intercepts. b. For exponential functions, focus on intercepts and end behavior. c. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.A1HS.24	Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions.
M.A1HS.25	Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square for a=1 only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions in exponential functions.

Cluster	Build a function that models a relationship between two quantities.
M.A1HS.26	Write linear, exponential, and quadratic functions that describe a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.
M.A1HS.27	Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship or given input-output pairs (include reading these from a table).

Cluster	**Build new functions from existing functions.**
M.A1HS.28	Identify the effect on the graphs of linear and exponential functions, f(x), with f(x) + k, and the graphs of quadratic functions, g(x), with g(x) + k, k g(x), g(kx), and g(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.A1HS.29	Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth.

Geometry

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.A1HS.30	Prove the slope criteria for parallel and perpendicular lines and use the slope criteria to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
M.A1HS.31	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.

Statistics and Probability

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.A1HS.32	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.A1HS.33	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.A1HS.34	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.A1HS.35	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association.

Cluster	**Interpret linear models.**
M.A1HS.36	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.
M.A1HS.37	Distinguish between correlation and causation.