West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstractly and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics - High School Geometry

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Basics of Geometry	Transformations and Congruence
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°). • Use coordinates and equations to describe geometric properties algebraically (e.g., write the equation for a circle in the plane with specified center and radius).	• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.
Similarity and Trigonometry	**Circles**
• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations (e.g., determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.).	• Apply theorems about circles to describe the relationships of components of a circle or formed by a circle and to find arc lengths and areas of sectors of circles.
Extending to Three Dimensions and Modeling	**Statistics and Probability**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Work with probability and using ideas from probability in everyday situations (e.g., compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Geometry:

Basics of Geometry	
Experiment with transformations in the plane.	Standard 1
Identify and utilize inductive and deductive reasoning.	Standard 2
Prove geometric theorems.	Standard 3
Use coordinates to prove simple geometric theorems algebraically.	Standard 4
Make geometric constructions.	Standard 5
Transformations and Congruence	
Experiment with transformations in the plane.	Standards 6-9
Understand congruence in terms of rigid motions.	Standards 10-13
Prove geometric theorems.	Standards 14-15
Use coordinates to prove simple geometric theorems algebraically.	Standard 16

Similarity and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 17-19
Prove theorems involving similarity.	Standards 20-21
Define trigonometric ratios and solve problems involving right triangles.	Standards 22-24
Apply trigonometry to general triangles.	Standards 25-27
Circles	
Understand and apply theorems about circles.	Standards 28-29
Find arc lengths and areas of sectors of circles.	Standard 30
Make geometric constructions.	Standards 31-33
Extending to Three Dimensions and Modeling	
Explain volume formulas and use them to solve problems.	Standards 34-35
Visualize the relation between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 36-37
Statistics and Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 38-42
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 43-46
Use probability to evaluate outcomes of decisions.	Standards 47-48

Basics of Geometry

Cluster	**Experiment with transformations in the plane.**
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Cluster	**Identify and utilize inductive and deductive reasoning.**
M.GHS.2	Construct and justify the validity of a logical argument. a. Identify the converse, inverse, and contrapositive of a conditional statement. b. Translate a short, verbal argument into symbolic form. c. Use Venn diagrams to represent set relationships. d. Use inductive and deductive reasoning.

Cluster	**Prove geometric theorems.**
M.GHS.3	Use appropriate methods of proof to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.4	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Cluster	**Make geometric constructions.**
M.GHS.5	Make formal geometric constructions with a variety of tools and methods, such as a compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.: a. copying a segment; b. copying an angle; c. bisecting a segment; d. bisecting an angle; e. constructing perpendicular lines, including the perpendicular bisector of a line segment; and f. constructing a line parallel to a given line through a point not on the line.

Transformations and Congruence

Cluster	**Experiment with transformations in the plane.**
M.GHS.6	Build on prior knowledge from rigid motions to: a. represent transformations using geometric concepts in the plane. b. describe transformations as functions that take points in the plane as inputs and give other points as outputs. c. compare transformations that preserve distance and angle to those that do not.
M.GHS.7	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
M.GHS.8	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
M.GHS.9	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Describe a sequence of transformations that will carry a given figure onto another.

Cluster	**Understand congruence in terms of rigid motions.**
M.GHS.10	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
M.GHS.11	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
M.GHS.12	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
M.GHS.13	Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	**Prove geometric theorems.**
M.GHS.14	Use appropriate methods of proof to prove theorems about triangles and lines. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
M.GHS.15	Use appropriate methods of proof to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.16	Use coordinates to prove simple geometric theorems about right triangles, quadrilaterals, and circles algebraically (e.g., derive the equation of a circle of given center and radius using the Pythagorean Theorem).

Similarity and Trigonometry

Cluster	**Understand similarity in terms of similarity transformations.**
M.GHS.17	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.GHS.18	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.GHS.19	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	**Prove theorems involving similarity.**
M.GHS.20	Use appropriate methods of proof to prove theorems about triangles involving similarity. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
M.GHS.21	Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Use the Pythagorean Theorem and similarity criteria to derive and apply special right triangles to solve problems.

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**
M.GHS.22	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.GHS.23	Explain and use the relationship between the sine and cosine of complementary angles.
M.GHS.24	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	**Apply trigonometry to general triangles.**
M.GHS.25	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.GHS.26	Prove the Laws of Sines and Cosines extending the definitions of sine and cosine to obtuse angles.
M.GHS.27	Understand and apply the Law of Sines and the Law of Cosines to solve problems and to find unknown measurements in right and non-right triangles.

Circles

Cluster	**Understand and apply theorems about circles.**
M.GHS.28	Prove that all circles are similar.
M.GHS.29	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Cluster	**Find arc lengths and areas of sectors of circles.**
M.GHS.30	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Cluster	**Make geometric constructions.**
M.GHS.31	Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
M.GHS.32	Construct a tangent line from a point outside a given circle to the circle.
M.GHS.33	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Extending to Three Dimensions and Modeling

Cluster	**Explain volume formulas and use them to solve problems.**
M.GHS.34	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.GHS.35	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems, including how area and volume scale under similarity transformations.

Cluster	**Visualize the relation between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.**
M.GHS.36	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
M.GHS.37	Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. a. Apply concepts of density based on area and volume in modeling situations. b. Apply geometric methods to solve design problems to satisfy given constraints.

Statistics and Probability

Cluster	**Understand independence and conditional probability and use them to interpret data.**
M.GHS.38	Describe events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events.
M.GHS.39	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. Use this characterization to determine if they are independent.
M.GHS.40	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.GHS.41	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.GHS.42	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
M.GHS.43	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.
M.GHS.44	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model.
M.GHS.45	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A) P(B|A) = P(B)P(A|B) and interpret the answer in terms of the model.
M.GHS.46	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	Use probability to evaluate outcomes of decisions.
M.GHS.47	Use probabilities to make fair decisions.
M.GHS.48	Analyze decisions and strategies using probability concepts.