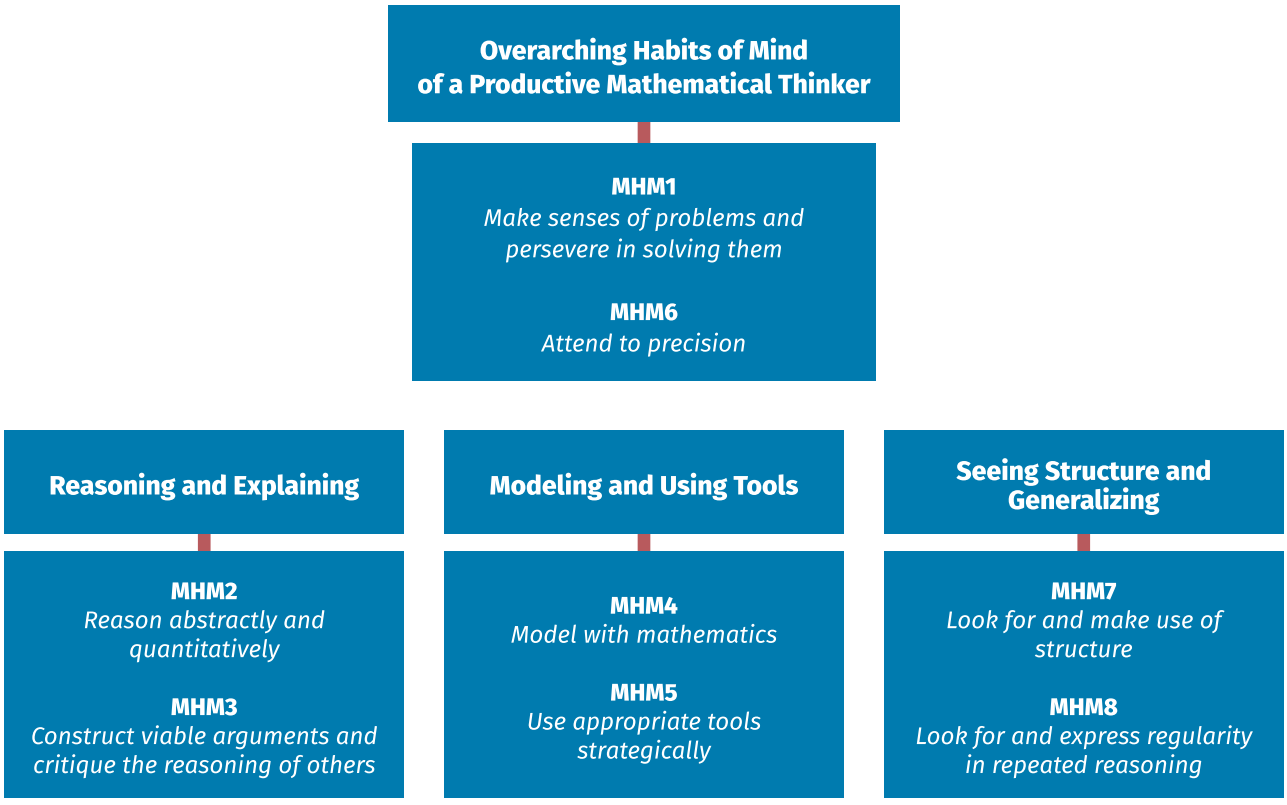
High School Algebra II - Mathematics III

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics - High School Algebra II – Mathematics III

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. The Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

The Number System	**Expressions and Equations**
• Extend the number system to complex numbers and perform arithmetic operations with complex numbers.	• Derive the formula for the sum of a geometric series, and use the formula to solve problems (e.g., calculate mortgage payments).
Functions	**Statistics and Probability**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Algebra II – Mathematics III:

The Number System	
Perform arithmetic operations with complex numbers.	Standards 1-2
Expressions and Equations	
Use complex numbers in polynomial identities and equations.	Standards 3-5
Interpret the structure of expressions.	Standards 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standards 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Solve systems of equations.	Standard 18
Functions	
Create equations that describe numbers or relationships.	Standards 19-21
Interpret functions that arise in applications in terms of a context.	Standards 22-23
Analyze functions using different representations.	Standards 24-26

Build a function that models a relationship between two quantities.	Standard 27
Build new functions from existing functions.	Standards 28-29
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 30
Statistics and Probability	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 31
Understand and evaluate random processes underlying statistical experiments.	Standards 32-33
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 34-37
Use probability to evaluate outcomes of decisions.	Standards 38-39

The Number System

Cluster	**Perform arithmetic operations with complex numbers.**
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b representing real numbers.
M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Expressions and Equations

Cluster	**Use complex numbers in polynomial identities and equations.**
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions.
M.A2HS.4	Factor special case polynomials with real coefficients that produce complex zeros.
M.A2HS.5	Show that the Fundamental Theorem of Algebra is true for quadratic polynomials with real coefficients.
Cluster	**Interpret the structure of expressions.**
M.A2HS.6	Interpret expressions including rational and polynomial expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
M.A2HS.7	Use the structure of expressions including polynomial and rational expressions to identify ways to rewrite them.
Cluster	**Write expressions in equivalent forms to solve problems.**
M.A2HS.8	Derive the formula for the sum of a finite geometric and use the formula to solve problems.

Cluster	Perform arithmetic operations on polynomials.
M.A2HS.9	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Perform operations on polynomials with degree higher than two.

Cluster	Understand the relationship between zeros and factors of polynomials.
M.A2HS.10	Apply the Remainder Theorem to polynomial functions.
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships.
M.A2HS.13	Apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n with coefficients determined, for example, by Pascal's Triangle.

Cluster	Rewrite rational expressions.
M.A2HS.14	Rewrite simple rational expressions in different forms; write a(x)/b(x) in different forms using inspection, long division, synthetic division, or, for the more complicated examples, a computer algebra system.
M.A2HS.15	Recognize that rational expressions form a system analogous to the rational numbers, namely, they are closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

Cluster	Represent and solve equations and inequalities graphically.
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, exponential, and logarithmic equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations).

Cluster	Solve systems of equations.
M.A2HS.18	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Cluster	**Create equations that describe numbers or relationships.**
M.A2HS.19	Create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems.
M.A2HS.20	Create equations in two or more variables, representing linear, exponential, and quadratic relationships, between quantities.
M.A2HS.21	Represent constraints by linear, exponential, or quadratic equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A2HS.22	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and where applicable, to the quantitative relationship it describes. • Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; and end behavior.
M.A2HS.23	Select a model function based on behavior of data and context to calculate and interpret the average rate of change of linear, exponential, quadratic, and model functions based on behavior of data and context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Cluster	**Analyze functions using different representations.**
M.A2HS.24	Graph quadratic, polynomial, square root, cube root, piecewise-defined functions, including step functions and absolute value functions, exponential, and logarithmic functions expressed symbolically and show key features of the graph. Use applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. a. For polynomial functions, focus on identifying zeros and showing end behavior. b. For exponential and logarithmic functions, focus on showing intercepts and end behavior. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.A2HS.25	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function focusing on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.26	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions. Focus on applications and how key features relate to characteristics of a situation.

Cluster	**Build a function that models a relationship between two quantities.**
M.A2HS.27	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations.

Cluster	**Build new functions from existing functions.**
M.A2HS.28	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations.
M.A2HS.29	Find inverse functions for simple polynomial, simple rational, simple radical, and use simple exponential functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Consider situations where the domain of the function must be restricted in order for the inverse to exist.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.A2HS.30	For exponential models, express as a logarithm the solution to $a \cdot b^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Statistics and Probability

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.A2HS.31	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Cluster	**Understand and evaluate random processes underlying statistical experiments.**
M.A2HS.32	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Compare theoretical and empirical results to evaluate the effectiveness.
M.A2HS.33	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Cluster	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**
M.A2HS.34	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

M.A2HS.35	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error using simulation models for random sampling. Informally develop the concepts of statistical significance and variability.
M.A2HS.36	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Recognize that some unlikely results can occur solely through randomness inherent in the system and "statistical significance" represents this likelihood. Make use of statistics as a way of dealing with, not eliminating, this inherent randomness.
M.A2HS.37	Evaluate reports based on data. Focus on data collection and how conclusions can be drawn from data.

Cluster	**Use probability to evaluate outcomes of decisions.**
M.A2HS.38	Use probabilities to make fair decisions, including situations involving quality control, false positive, and false negative results.
M.A2HS.39	Analyze decisions and strategies using probability concepts, including situations involving quality control, false positive, and false negative results.