Applied Statistics

West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstractly and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Applied Statistics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Applied Statistics provides authentic experiences in statistics designed to strengthen students' application of the statistical method. Students will conduct statistical simulations to model everyday situations in an increasingly data-rich world. Students in this course will select appropriate graphical and numerical methods to explore data, design and implement a plan to collect and analyze data, and use probability to evaluate outcomes and make decisions. Students will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, radical, and rational functions. Students will use multiple representations, technology, applications and modeling in problem-solving contexts. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Exploring Data	Designing Studies
• Represent data visually and calculate statistical measures that describe the data set (e.g., construct and interpret a histogram for a student created data set).	• Design a plan to collect data using an appropriate sampling method to solve a problem (e.g., design and conduct an experiment to determine the effect of a treatment).
Functions and Modeling	**Probability and Informed Decisions**
• Explore expressions, functions, and models to highlight key features that provide insight into their structure, properties, and form (e.g., model situations using mathematics to provide an optimal solution).	• Make inferences and justify conclusions from data and analyze decisions and strategies using probability concepts (e.g., compare experimental and theoretical probabilities to make informed decisions).

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Applied Statistics:

Exploring Data	
Select appropriate graphical and numerical methods to explore data.	Standards 1-7
Designing Studies	
Design and implement a plan to collect and analyze data.	Standards 8-12
Functions and Modeling	
Explore expressions, functions, and models to describe numbers or relationships.	Standards 13-21
Probability and Informed Decisions	
Use probability to evaluate outcomes and make decisions.	Standards 22-29

Exploring Data

Cluster	Select appropriate graphical and numerical methods to explore data.
M.ASHS.1	Generate appropriate ways to display various types of data. Instructional Note: Build on data displays introduced in prior courses.
M.ASHS.2	Calculate appropriate measures of center, variability, and position for data. Instructional note: Include comparisons of mean vs. median, standard deviation vs. IQR.
M.ASHS.3	Use graphical displays and summary statistics to make conclusions. Informally develop the concept of statistical significance; a result that is unlikely to have occurred by chance alone. Instructional Note: Focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.ASHS.4	Represent data in two variables to model relationships between quantities. Instructional Note: Students will use multiple representations with appropriate labels and scales.
M.ASHS.5	Select a function that models a relationship between two quantities and interpret key features of graphs and tables in terms of the quantities. Instructional Note: Focus on form, strength, direction, and departures from a model based on data and context.
M.ASHS.6	Compare characteristics of two data sets each represented in different ways (algebraically, graphically, numerically, and verbally). Instructional Note: Focus on applications and how key features relate to characteristics of a situation and select an appropriate model.
M.ASHS.7	Use appropriate measures of center and spread to describe a distribution. Instructional Note: Emphasize that only some data are well described by a normal distribution.

Designing Studies

Cluster	Design and implement a plan to collect and analyze data.
M.ASHS.8	Develop a process for making inferences about population parameters based on a random sample through data collection and analysis.
M.ASHS.9	Evaluate the results from a given data-generating process to determine consistency between theoretical and experimental probabilities. Instructional Note: Include the Law of Large Numbers.
M.ASHS.10	Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain the importance of randomization in each method. Instructional Note: Emphasize that the way in which data is collected determines the scope and nature of the conclusions.
M.ASHS.11	Use data from a sample survey to estimate a population mean or proportion. Instructional Note: Develop the connection between sample size and margin of error.
M.ASHS.12	Design and conduct an experiment to compare two treatments. Instructional Note: Include randomization, replication, blocking, and control in the design.

Cluster	**Explore expressions, functions, and models to describe numbers or relationships.**
M.ASHS.13	Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational functions, and use them to solve problems.
M.ASHS.14	Develop the concept of a complex number i such that $i^2 = -1$. Understand that every complex number can be written in the form a + bi with a and b real.
M.ASHS.15	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
M.ASHS.16	Use the structure of polynomial and rational expressions to identify ways to rewrite them.
M.ASHS.17	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
M.ASHS.18	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.
M.ASHS.19	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.
M.ASHS.20	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (e.g., solve $z = \frac{x-\mu}{\sigma}$ for σ and Margin of Error = $z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ for *n*). Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.
M.ASHS.21	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative extrema; symmetries; and end behavior. Instructional Note: Emphasize the selection of a model function based on the behavior of data in context.

Cluster	**Use probability to evaluate outcomes and make decisions.**
M.ASHS.22	Connect sampling variability and margin of error to generate and interpret plausible parameter values. Instructional Note: The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred by chance alone. Focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.ASHS.23	Interpret results from a randomized experiment comparing two treatments. Use simulations to decide if experimental results are significant. Instructional Note: Develop informally the comparison of an observed result and an established probability value (for example $p \leq 0.05$).
M.ASHS.24	Evaluate claims based on data reports. Instructional Note: Data reports can be gathered from media.
M.ASHS.25	Use probability rules to make fair decisions. Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.26	Use two-way tables, tree diagrams, Venn diagrams, or 10 x 10 grids to model probabilities.
M.ASHS.27	Justify a decision using probability rules (e.g., product testing, medical testing, weather forecasting, marketing, or sports coaching decisions). Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.28	Perform appropriate calculations for given outcomes and decisions based on expected values for non-normal distributions. Instructional Note: Focus on uniform, discrete, continuous (geometric areas), or games of chance.
M.ASHS.29	Given data from a normal distribution, use the mean and standard deviation to estimate population percentages. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Recognize that there are data sets for which such a procedure is not appropriate. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities).