Transition Mathematics for Seniors

West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstractly and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Transition Mathematics for Seniors

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Transition Mathematics for Seniors prepares students for their entry-level, credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates and to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Transition Mathematics for Seniors:

Number and Quantity – The Real Number System	
Reason quantitatively and use units to solve problems.	Standards 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra – Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standard 5
Understand the connections between proportional relationship, lines, and linear equations.	Standards 6-8

Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 9
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 10-13
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 14
Solve equations and inequalities in one variable.	Standards 15-17
Solve systems of equations.	Standards 18-20
Represent and solve equations and inequalities graphically.	Standards 21-22
Functions – Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 23
Interpret functions that arise in applications in terms of the context.	Standards 24-27
Analyze functions using different representations.	Standards 28-34
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 35-36
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 37-38
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 39-40
Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 41
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 42-45
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 46-49
Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 50

Number and Quantity – The Real Number System

Cluster	**Extend the properties of exponents to rational exponents.**
M.TMS.1	Use units to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Number and Quantity – The Complex Number System

Cluster	**Use complex numbers in polynomial identities and equations.**
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.

Algebra – Seeing Structure in Expressions

Cluster	**Interpret the structure of expressions.**
M.TMS.4	Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Cluster	**Understand the connections between proportional relationship, lines, and linear equations.**
M.TMS.6	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways.
M.TMS.7	Explain (e.g., using similar triangles) why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
M.TMS.8	Solve linear equations in one variable.

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	**Perform arithmetic operations on polynomials.**
M.TMS.9	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Algebra – Creating Equations

Cluster	**Create equations that describe numbers or relationships.**
M.TMS.10	Create equations and inequalities in one variable, representing linear, exponential, quadratic, and simple rational relationships, and use them to solve problems.
M.TMS.11	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.12	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
M.TMS.13	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Algebra – Reasoning with Equations and Inequalities

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.TMS.14	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Cluster	**Solve equations and inequalities in one variable.**
M.TMS.15	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.TMS.16	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.TMS.17	Solve quadratic equations in one variable by inspection (e.g., $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Recognize the concept of complex solutions when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b.

Cluster	**Solve systems of equations.**
M.TMS.18	Understand and demonstrate ways to manipulate a system of two equations in two variables while preserving its solution set.
M.TMS.19	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Instructional Note: Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.TMS.20	Explain why the x-coordinates of the points where the graphs of the linear, polynomial, rational, absolute value, and exponential equations y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations).

Cluster	**Represent and solve equations and inequalities graphically.**
M.TMS.21	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
M.TMS.22	Graph the solutions to a linear inequality in two variables as a half-plane graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**
M.TMS.23	Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

Cluster	**Interpret functions that arise in applications in terms of the context.**
M.TMS.24	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
M.TMS.25	Interpret the parameters in a linear or exponential function in terms of a context.
M.TMS.26	Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.
M.TMS.27	Distinguish between situations that can be modeled with linear functions and with exponential functions.

Cluster	Analyze functions using different representations.
M.TMS.28	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
M.TMS.29	Describe qualitatively the functional relationship between two quantities by analyzing a graph.
M.TMS.30	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
M.TMS.31	Graph linear, quadratic, and polynomial functions expressed symbolically and show key features of the graph. a. For linear functions, focus intercepts. b. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form. c. For polynomial functions, focus on identifying zeros and showing end behavior. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.TMS.32	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.
M.TMS.33	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M.TMS.34	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions.

Functions - Building Functions

Cluster	Build a function that models a relationship between two quantities.
M.TMS.35	Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table).
M.TMS.37	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Geometric Measuring and Dimension

Cluster	Explain volume formulas and use them to solve problems.
M.TMS.37	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.TMS.38	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Geometry – Expressing Geometric Properties with Equations

Cluster	Use coordinates to prove simple geometric theorems algebraically
M.TMS.39	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.40	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.

Geometry – Modeling with Geometry

Cluster	Apply geometric concepts in modeling situations.
M.TMS.41	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.TMS.42	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Interpret linear models.
M.TMS.43	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.

M.TMS.44	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.TMS.45	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.TMS.46	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.TMS.47	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.TMS.48	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.TMS.49	Distinguish between correlation and causation.

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	**Understand and evaluate random processes underlying statistical experiments.**
M.TMS.50	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.