Introduction to Mathematical Applications

West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstractly and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important “processes and proficiencies “ with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics’ process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck’s research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a “growth mindset.” In Dweck’s estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Introduction to Mathematical Applications

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Introduction to Mathematical Applications will solidify their quantitative literacy by enhancing numeracy and problem-solving skills as they investigate and use fundamental concepts of algebra, geometry, and statistical analysis to apply to authentic career projects and scenarios. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity	**Algebra: Seeing Structure in Expressions**
• Develop an understanding of basic operations, equivalent representations, and properties of the real number system.	• Create equations or inequalities that model physical situations.
Functions: Interpreting Functions	**Geometry/Trigonometry**
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent, and interpret functions and their applications.	• Solve application problems by calculating area or surface area in a two-dimensional object or volume in three-dimensional objects. • Understand and apply the Pythagorean Theorem for solving real-world problems (e.g., checking accuracy on gate construction, conduit bending).
Modeling	**Statistics: Interpreting Categorical & Quantitative Data**
• Create and use two- and three-dimensional representations of authentic situations in problem solving. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Analyze and interpret tables, charts, and graphs (e.g., interpret a body mass index (BMI) chart). • Distinguish between correlation and causation.
Finance Mathematics	
• Determine, represent, and analyze mathematical models for personal finance.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Introduction to Mathematical Applications:

Number and Quantity	
Mathematics as a language.	Standards 1-3
Mathematics and Measurement.	Standards 4-7
The Real Number System.	Standards 8-9
Algebra: Seeing Structure in Expressions	
Understand the connections between proportional relationships, lines, and linear equations.	Standards 10-11
Create equations that describe numbers or relationships.	Standards 12-15
Solve systems of equations.	Standard 16

Functions: Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 17
Analyze functions using different representations.	Standards 18-20
Build a function that models a relationship between two quantities.	Standards 21-22
Geometry/Trigonometry	
Visualize relationships between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 23-24
Use geometric theorems and formulas to solve problems.	Standards 25-29
Modeling	
Concrete geometric representation (physical modeling).	Standards 30-31
Summarize, represent, and interpret data on two quantitative variables.	Standards 32-34
Statistics: Interpreting Categorical & Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 35-39
Finance Mathematics	
Understand financial models.	Standards 40-41
Personal use of finance.	Standards 42-43

Number and Quantity

Cluster	Math as a language
M.IMA.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyzing the soundness of mathematical arguments of others.
M.IMA.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.
M.IMA.3	Use units to understand problems and to guide the solutions of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Cluster	Math and Measurement.
M.IMA.4	Select and correctly use an appropriate tool (e.g., tape measure, ruler, compass, level, micrometer, scale, protractor, thermometer, speedometer, odometer, pressure gauge, measuring squares, multimeter) to measure and/or calculate lengths, distances, directions, masses, temperatures, rates of change (e.g., slope, speed), areas, volumes, voltages, currents, and resistances.
M.IMA.5	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

M.IMA.6	Solve real-world problems requiring conversion of units using dimensional analysis for measurements in English and metric systems. Solve problems involving multiple units of measurement (e.g., converting between currencies, calculating dosages of medicine, trip planning from miles to kilometers).
M.IMA.7	Distinguish between proportional and non-proportional situations, apply proportional reasoning when appropriate, solve for an unknown quantity in proportional situations; apply scale factors to perform indirect measurements using maps, blueprints, concentrations, dosages, and densities.

Cluster	**The Real Number System.**
M.IMA.8	Perform operations and convert quantities between fractions, decimals, and percents using positive and negative numbers, fractions, absolute value, decimals, percentages, and scientific notation (e.g., given the cost of a project, determine what percentage of the budget were salaries; percent of increase/decrease).
M.IMA.9	Solve real-world problems in a variety of contexts by representing quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as scientific notation and the meaning of a fraction or percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages.

Algebra – Seeing Structure in Expressions

Cluster	**Understand the connections between proportional relationships, lines, and linear equations.**
M.IMA.10	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed (e.g., labor cost per time, material cost per job).
M.IMA.11	Solve application problems using direct and inverse variation equations (e.g., determine the mechanical advantage of gears, Ohm's Law).

Cluster	**Create equations that describe numbers or relationships.**
M.IMA.12	Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem.

M.IMA.13	Analyze real-world problem situations and use variables to construct and solve equations and inequalities in one variable, representing linear, exponential, and simple rational functions (e.g., using spreadsheet functions, determine sale price of items).
M.IMA.14	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales (e.g., profit vs. number of units, cost vs. number of units, resistance vs. current).
M.IMA.15	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (e.g., rearrange Ohm's law V = IR to highlight resistance R).

Cluster	**Solve systems of equations.**
M.IMA.16	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables (e.g., childcare facility – sq. footage to number of children; solving electrical current in a circuit with multiple paths, break-even point).

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**
M.IMA.17	Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

Cluster	**Analyze functions using different representations.**
M.IMA.18	Interpret the parameters in a linear function in terms of a context.
M.IMA.19	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
M.IMA.20	Describe qualitatively the functional relationship between two quantities by analyzing a graph.

Cluster	**Build a function that models a relationship between two quantities.**
M.IMA.21	Represent application problems as linear equations. Write a function that describes a relationship between two quantities (e.g., level of education versus pay; rate of speed versus fuel consumption; caloric intake versus expenditure).
M.IMA.22	Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.

Geometry/Trigonometry

Cluster	Visualize relationships between two-dimensional and three-dimensional objects and apply geometric concepts in modeling situations.
M.IMA.23	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects (e.g., three-view drawings and blueprints).
M.TTMS.24	Use two- and three-dimensional shapes and circles, their measures, and their properties to describe objects. a. Apply concepts of density based on area and volume in modeling situations. b. Apply geometric methods to solve design problems to satisfy given constraints (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Cluster	Use geometric theorems and formulas to solve problems.
M.IMA.25	Explore theorems about triangles to solve real-world application problems.
M.IMA.26	Understand and apply the Pythagorean Theorem for solving real-world problems (e.g., checking accuracy on gate construction, conduit bending, roof pitch). Instructional Note: If students have had experience and exposure to right triangle trigonometry, extend this standard to apply trigonometric ratios and special right triangle relationships, 30-60-90 and 45-45-90, to solve right triangles in applied problems.
M.IMA.27	Solve application problems by calculating area and surface area for two-dimensional objects (e.g., calculate the cost of installing flooring in a building and painting the interior and exterior of a building based on square footage).
M.IMA.28	Solve application problems by calculating volume for three-dimensional objects using formulas for cylinders, pyramids, prisms, cones, and spheres (e.g., compute amount of cement needed for a sidewalk, amount of water in a fire hose, amount of air in ductwork).
M.IMA.29	Solve application problems by calculating circumference, area, radius, diameter, area of sector, arc length of a circle with appropriate unit labels (e.g., develop a circular watering system).

Modeling

Cluster	Concrete geometric representation (physical modeling) .
M.IMA.30	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.IMA.31	Gather data, conduct investigations, and apply mathematical concepts and models to solve problems (e.g., designing and building a house or a car).

Cluster	Summarize, represent, and interpret data on two quantitative variables.
M.IMA.32	Collect numerical bivariate data; represent data on two quantitative variables on a scatter plot; determine whether or not a relationship exists; if so, describe how the variables are related and select a function to model the data, justify the selection and use the model to make predictions (e.g., cost of the materials for a construction project, cost of the labor for a project, cost and value of a vehicle based on depreciation).
M.IMA.33	For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. Interpret the rate of change and the constant term of a linear model in the context of the data.
M.IMA.34	Identify positive and negative correlations (e.g., vehicle depreciation). Use technology to compute and interpret the correlation coefficient of a linear fit. Instructional Note: The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

Statistics and Probability: Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.IMA.35	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.IMA.36	Analyze and interpret tables, charts, and graphs (e.g., interpret a body mass index (BMI) chart).
M.IMA.37	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.IMA.38	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.IMA.39	Distinguish between correlation and causation.

Finance Mathematics

Cluster	Understanding financial models.
M.IMA.40	Determine, represent, and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., business loans, auto, mortgage, and/or credit card).
M.IMA.41	Determine, represent, and analyze mathematical models for investments involving simple and compound interest with and without additional deposits (e.g., savings accounts, bonds, and/or certificates of deposit).

Cluster	Personal use of finance.
M.IMA.42	Research, develop, and analyze personal budgets based on given parameters (e.g., fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission, career choice, geographic region, retirement, and/or investment planning).
M.IMA.43	Research and analyze taxes including payroll, sales, personal property, real estate, and income tax returns.