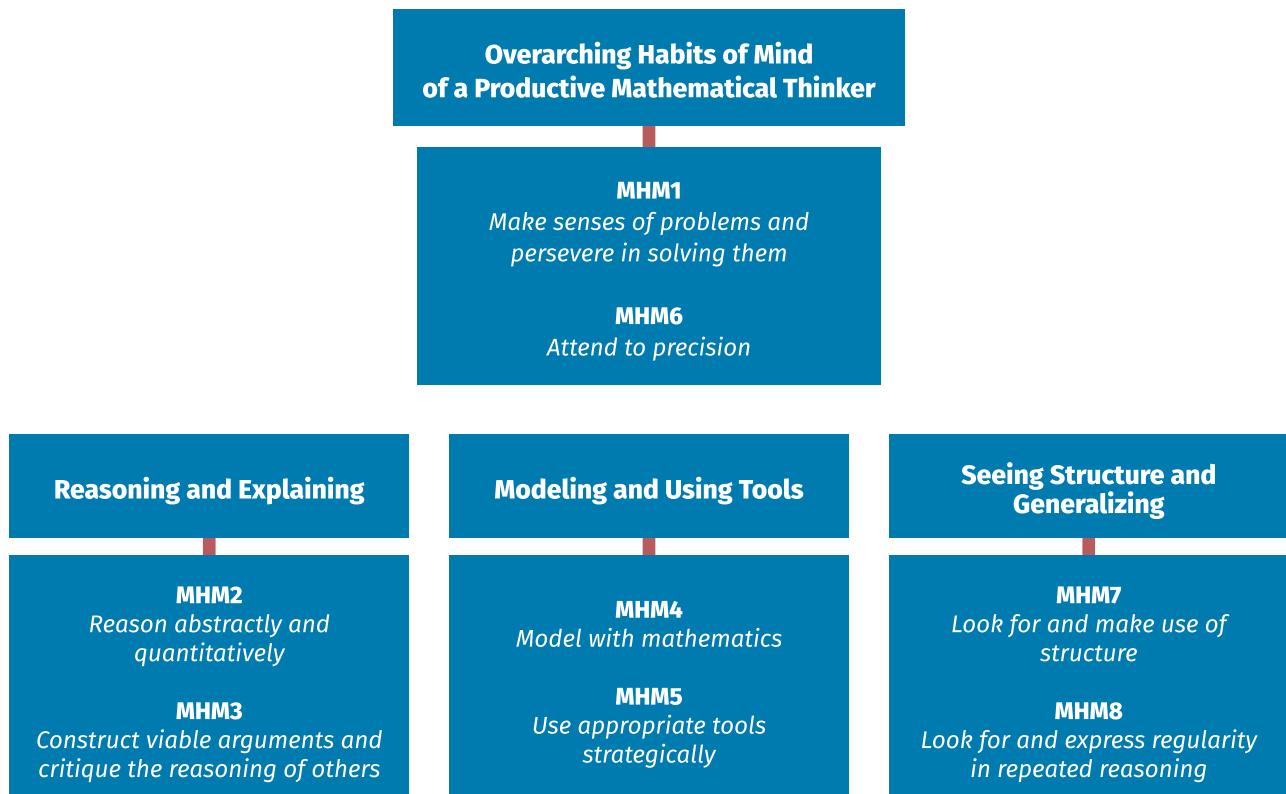


## Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students’ broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

### Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important “processes and proficiencies “ with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics’ process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck’s research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a “growth mindset.” In Dweck’s estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

# Policy 2520.2B

## *West Virginia College- and Career-Readiness Standards for Mathematics*

### **Mathematical Habits of Mind**

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

#### **MHM1. Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### **MHM2. Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

#### **MHM3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

#### **MHM4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **MHM5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **MHM6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**MHM7. Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**MHM8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$  and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Mathematics – Statistics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Knowledge of topics related to probability and statistics is critical to decision-making and to the analysis of data. The Statistics course is designed to be a full-year course that addresses all standards and provides an opportunity to address the fundamental ideas and most commonly used techniques to organize and make sense of data. This course builds on knowledge of probability, randomness, and variability to provide students with an understanding of experimental design, estimation, hypothesis testing, and effective communication of experimental results. Using concepts of probability and statistics, students predict the likelihood of an event occurring, organize and evaluate data, and identify the significance of statements. Students investigate types of probability, determine probability and odds using multiple counting principles and distributions, and apply the concepts to real-world problems. Statistical information collected and analyzed by students is used to investigate ways of collecting, displaying, and analyzing data. Students analyze and justify using statistical concepts to test validity of a hypothesis and of correlation as applied in real-world situations. Students identify a real-world situation that involves statistical concepts, make a hypothesis as to the outcome; develop, justify, and implement a method to collect, organize, and analyze data; generalize the results to make a conclusion, compare the hypothesis and the conclusion; present their findings using predictive and analytic tools in a clear and concise manner. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students

will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

<p><b>Descriptive Statistics</b></p> <ul style="list-style-type: none"> <li>Given a two-way table of relative frequencies that summarizes survey data, relate a person's highest level of education and their role model to determine if a person whose highest level of education is a bachelor's degree is more likely to have a family member than a stranger as a role model.</li> </ul>	<p><b>Probability</b></p> <ul style="list-style-type: none"> <li>A cereal company is putting a prize in each box of cereal. The company is offering four different and evenly distributed prizes. How many boxes should one expect to need to buy to get all four prizes?</li> </ul>
<p><b>Probability Distributions</b></p> <ul style="list-style-type: none"> <li>The heights of five women are measured to be 63 inches, 68 inches, 56 inches, 64 inches, and 67 inches. Determine the expected value of the height of a randomly chosen woman.</li> </ul>	<p><b>Correlation and Regression</b></p> <ul style="list-style-type: none"> <li>A random sample of beef hotdogs was taken and the amount of sodium (in mg) and calories were measured. Use the provided information and the regression equation to determine the amount of sodium in a beef hotdog with 170 calories and in a beef hotdog with 120 calories. Which of the calculated sodium levels is closer to the true sodium level? Why?</li> </ul>
<p><b>Confidence Intervals</b></p> <ul style="list-style-type: none"> <li>Alyssa has over 500 songs saved on her phone. She wants to estimate the proportion of songs by a female artist. After taking a simple random sample of 50 songs, she finds that 20 of the sampled songs are by a female artist. Determine a 99% confidence interval for the proportion of songs on her phone that are by a female artist.</li> </ul>	<p><b>Hypothesis Testing with One Variable</b></p> <ul style="list-style-type: none"> <li>Ellen has a pair of dogs and she noticed that they seem to breed more male puppies than female puppies. In the next litter of 12 puppies, there were 9 male puppies. Test the hypothesis that each puppy has an equal chance of 50% of being either male or female versus the alternative that the chance of a male puppy is greater. Look at the results of 1000 simulations, each simulating 12 puppies with a 50% chance of being male or female. According to the simulations, what is the probability of having 9 male puppies or more out of 12? If the observed outcome has a probability less than 1% under the tested hypotheses, reject the hypothesis. What should be concluded regarding the hypothesis?</li> </ul>
<p><b>Statistical Inference</b></p>	
<ul style="list-style-type: none"> <li>A researcher wants to know if the children from three schools have equal mean IQ scores. Each school enrolls 1000 students. But there is neither the time nor funding to test all 3000 students. Based on a simple random survey of 10 students from each school, perform and analyze an ANOVA test.</li> </ul>	



## Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Statistics:

<b>Descriptive Statistics</b>	
Summarize, represent, and interpret data on single count or measurement variable.	Standards 1-8
<b>Probability</b>	
Understand independence and conditional probability and use them to interpret data.	Standards 9-17
<b>Probability Distributions</b>	
Calculate expected values and use them to solve problems.	Standards 18-24
<b>Correlation and Regression</b>	
Interpret linear models.	Standards 25-27
<b>Confidence Intervals</b>	
Determine and interpret confidence intervals.	Standards 28-33
<b>Hypothesis Testing with One Variable</b>	
Use hypothesis testing in making and interpreting decisions.	Standards 34-42
<b>Statistical Inference</b>	
Determine and use correlation.	Standards 43-46
Use linear regression to predict and interpret.	Standards 47-51
Use statistical tests to determine a relationship.	Standards 52-57

<b>Cluster</b>	<b>Summarize, represent, and interpret data on single count or measurement variable.</b>
M.PS.1	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
M.PS.2	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
M.PS.3	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
M.PS.4	Evaluate reports based on data. Write a function that describes a relationship between two quantities.
M.PS.5	Represent data with plots on the real number line (dots plots, histograms, and box plots).
M.PS.6	Use statistics appropriate to the shape of the data distributions to compare center and spread of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.PS.7	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.PS.8	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.



Cluster	<b>Understand independence and conditional probability and use them to interpret data.</b>
M.PS.9	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
M.PS.10	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.
M.PS.11	Recognize the conditional probability of A given B as $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.PS.12	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
M.PS.13	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
M.PS.14	Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and interpret the answer in terms of the model.
M.PS.15	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and interpret the answer in terms of the model.
M.PS.16	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$ and interpret the answer in terms of the model.
M.PS.17	Use permutations and combinations to compute probabilities of compound events and solve problems.

## Probability Distributions

<b>Cluster</b>	<b>Calculate expected values and use them to solve problems.</b>
M.PS.18	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
M.PS.19	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.PS.20	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated to find the expected value.
M.PS.21	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically to find the expected value.
M.PS.22	Weight the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values (e.g., find the expected payoff for a game of chance).
M.PS.23	Evaluate and compare strategies on the basis of expected values.
M.PS.24	Analyze decisions and strategies using probability concepts.

## Correlation and Regression

<b>Cluster</b>	<b>Interpret linear models.</b>
M.PS.25	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. <ol style="list-style-type: none"><li>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</li><li>Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate.</li><li>Fit a linear function for scatter plots that suggest a linear association.</li></ol>
M.PS.26	Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit. Instructional Note: The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.PS.27	Distinguish between correlation and causation.

## Confidence Intervals

<b>Cluster</b>	<b>Determine and interpret confidence intervals.</b>
M.PS.28	Find the point estimate and margin of error in a given scenario.
M.PS.29	Construct and interpret confidence intervals for the population mean.
M.PS.30	Determine minimum sample size requirements when estimating mean, $\mu$ (population proportion).
M.PS.31	Interpret the t-distribution and use t-distribution table in real-world scenarios.
M.PS.32	Construct confidence intervals when the sample size, $n$ , is less than 30, population is normally distributed, and standard deviation, $\sigma$ , is unknown.
M.PS.33	Interpret the chi-square distribution and use chi-square distribution table. Use the chi-square distribution to construct a confidence interval for the variance and standard deviation.

## Hypothesis Testing with One Variable

<b>Cluster</b>	<b>Use hypothesis testing in making and interpreting decisions.</b>
M.PS.34	Interpret a hypothesis test; state a null hypothesis and an alternative hypothesis.
M.PS.35	Identify Type I and Type II errors and interpret the level of significance.
M.PS.36	Use one-tailed and two-tailed statistical tests to find p-value.
M.PS.37	Make and interpret decisions on comparing two hypotheses based on results of a statistical test. Write a claim for a hypothesis test.
M.PS.38	Find probability values and test for mean. Use in a z-test.
M.PS.39	Find critical values and rejection regions in a normal distribution. Use rejection regions for a z-test.
M.PS.40	Find critical values in a t-distribution and use the t-test to test a mean.
M.PS.41	Use the z-test to test a population proportion, $p$ .
M.PS.42	Find critical values for chi squared test. Use the chi squared test to test a variance or a standard deviation.

## Statistical Inference

<b>Cluster</b>	<b>Determine and use correlation.</b>
M.PS.43	Find a correlation coefficient.
M.PS.44	Test a population correlation coefficient using a table.
M.PS.45	Perform a hypothesis test for a population correlation coefficient.
M.PS.46	Distinguish between correlation and causation.

<b>Cluster</b>	<b>Use linear regression to predict and interpret.</b>
M.PS.47	Find the equation of a regression line; predict y-values using a regression line.
M.PS.48	Interpret the types of variation about a regression line.
M.PS.49	Find and interpret the coefficient of determination.
M.PS.50	Find and interpret the standard error of estimate for a regression line; construct and interpret a prediction interval for y.
M.PS.51	Use technology to find a multiple regression equation, the standard error of estimate, and the coefficient of determination.

<b>Cluster</b>	<b>Use statistical tests to determine a relationship.</b>
M.PS.52	Use a contingency table to find expected frequencies.
M.PS.53	Use the chi-squared distribution to test whether a frequency distribution fits a claimed distribution and to test whether two variables are independent.
M.PS.54	Interpret the F-distribution and use an F-table to find critical values.
M.PS.55	Perform a two-sample F-test to compare two variances.
M.PS.56	Perform a two-sample F-test to compare two variances. Interpret the F-distribution and use an F-table to find critical values.
M.PS.57	Use one-way analysis of variance to test claims involving three or more means. Introduce two-way analysis of variance.

