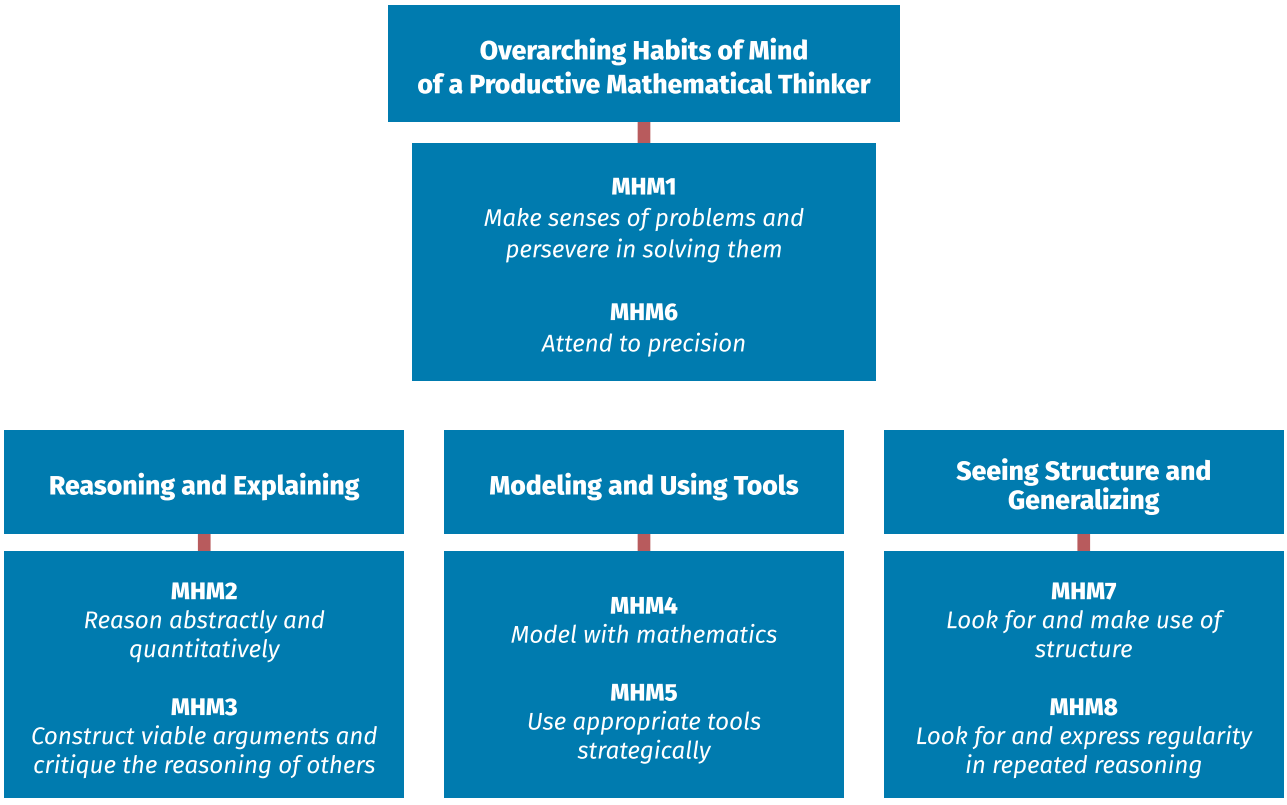
Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Data Science

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Data Science provides students an opportunity to integrate mathematics, statistics, and computer science to analyze and interact with data. Data Science is an interdisciplinary field where mathematical principles and scientific methods are applied to datasets using technology and computing skills to solve problems. It relies heavily on the mathematical and statistical reasoning that is developed during prior courses and contextualizes those concepts with computational solutions. Students will engage with data through cycles of exploration, visualization, analysis, communication, and application. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Explore Data	Visualize Data
• Develop questions that can be answered by exploring multivariable data sets. • Explore implications of complex data sets, collection methods, and privacy concerns.	• Use technology to generate a visualization appropriate to the data set to explore the data and generate questions.
Analyze Data	**Communicate Using Data**
• Choose appropriate statistics, understanding how algorithms can be used to calculate these values. • Use the shape of distributions to determine probabilities of situations leading to decision-making. • Fit multiple regression models; compare the strength of each model; and use the appropriate regression model to make predictions.	• Represent findings to a particular audience using the relevant interdisciplinary language of mathematics and data science.
Application of Data Science	
• Utilize a coding language to store, analyze, and model with data. • Describe the application of artificial intelligence, machine learning, and natural language processing to data science. • Complete a capstone project using techniques learned throughout the course.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Data Science:

Explore Data	
Ask and develop questions; collect data; and consider ethics and bias.	Standards 1-5
Research issues, access multivariable data, and clean data.	Standards 6-9
Visualize Data	
Display data.	Standards 10-15
Analyze Data	
Choose appropriate statistical values.	Standards 16-17
Fit bivariate data to functions using regression.	Standards 18-20
Understand the use of algorithms in statistical tests.	Standards 21-22
Understand probability in relation to decision-making.	Standards 23-24
Communicate Using Data	
Compare distributions.	Standards 25-26
Evaluate claims.	Standards 27-28
Report conclusions in multiple formats.	Standard 29
Applications of Data Science	
Understand security and ethics.	Standards 30-31
Explore artificial intelligence.	Standards 32-35
Utilize a coding language.	Standards 36-41
Apply data science to a capstone project.	Standard 42

Explore Data

Cluster	**Ask and develop questions; collect data; and consider ethics and bias.**
M.DSHS.1	Describe techniques for locating and collecting small- and large-scale data sets.
M.DSHS.2	Recognize a question that can be explored or answered using data science, including statistical questions.
M.DSHS.3	Use technology to informally describe the shape, variability, and center of a distribution of data.
M.DSHS.4	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results and evaluate a variety of resources used to collect data for accuracy, perspective, credibility, relevance, and privacy concerns.
M.DSHS.5	Understand that random sampling tends to produce representative samples that support valid inferences and generalizations about a population.

Cluster	**Research issues, access multivariable data, and clean data.**
M.DSHS.6	Explore and understand real-world issues and problems using multivariable data sets to hypothesize solutions.
M.DSHS.7	Access data from a variety of sources and apply mathematical concepts and models to solve problems in mathematics and other disciplines.
M.DSHS.8	Using programming techniques and spreadsheet capabilities, clean, store, analyze, and model with data sets.
M.DSHS.9	Compare techniques (e.g., sorting, statistics, searching) for analyzing multivariable data sets.

Visualize Data

Cluster	**Display data.**
M.DSHS.10	Use appropriate tools to represent data visually.
M.DSHS.11	Use appropriate tools and multiple representations to represent and model relationships of quantitative multivariable data consisting of at least four variables.
M.DSHS.12	Describe visual patterns in quantitative data such as clustering, outliers, positive or negative association, linear association, and nonlinear association (e.g., determine form, strength, and direction).
M.DSHS.13	Visualize categorical data using appropriate models such as mosaic plots, stacked bar graphs, etc. Recognize possible associations and data trends.
M.DSHS.14	Use methods of geospatial analysis to graphically or spatially represent natural phenomena.
M.DSHS.15	Understand the use of simulation to compare probabilities from a model to observed frequencies; explain possible sources of discrepancy.

Cluster	**Choose appropriate statistical values.**
M.DSHS.16	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.DSHS.17	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Cluster	**Fit bivariate data to functions using regression.**
M.DSHS.18	Use technology to create a regression for data that suggests a linear association. Compute the correlation coefficient, coefficient of determination, and residual plot, and interpret the results in the context of the problem.
M.DSHS.19	Fit a function to the data that does not suggest a linear association; use algebraic re-expression of the function to fit the data to solve problems in the context of the situation.
M.DSHS.20	Interpret key features such as intercepts, rate of change, and turning points of models in the context of the data.

Cluster	**Understand the use of algorithms in statistical tests.**
M.DSHS.21	Examine existing algorithms and describe connections to algebraic and statistical functions, sets, and logic.
M.DSHS.22	Develop algorithms in order to solve mathematical problems.

Cluster	**Understand probability in relation to decision-making.**
M.DSHS.23	Use the concepts of independent events and conditional probabilities to calculate and interpret outcomes of chance events to make data-informed decisions. Recognize and explain the concepts of conditional probability and independence to multiple audiences and contexts.
M.DSHS.24	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, using the area beneath the curve to make estimations of frequencies. Instructional Note: Emphasize that only some data are well described by a normal distribution.

Communicate Using Data

Cluster	**Compare distributions.**
M.DSHS.25	Assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (e.g., The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability on either team; on a dot plot, the separation between the two distributions of heights is noticeable).
M.DSHS.26	Analyze and communicate the benefits and limitations of data visualization tools to solve a real-world problem.

Cluster	**Evaluate claims.**
M.DSHS.27	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.
M.DSHS.28	Evaluate claims based on data reports gathered from a variety of sources such as the media, scientific journals, census data, etc.

Cluster	**Report conclusions in multiple formats.**
M.DSHS.29	Report results using an appropriate format (digital presentation, verbal, textual, etc.) and to a particular audience using the relevant language of mathematics and data science. Use data displays and interpret results in terms of the question studied.

Application of Data Science

Cluster	**Understand security and ethics.**
M.DSHS.30	Explore various legal and ethical standards for data ownership and the implications of the standards to the study and application of data science.
M.DSHS.31	Describe and understand how data is collected from both individuals and groups of individuals, shared, and used.

Cluster	Explore artificial intelligence.
M.DSHS.32	Know and identify examples of real-world or societal machine learning applications.
M.DSHS.33	Describe basic machine learning concepts such as training a model and evaluating model performance.
M.DSHS.34	Know and identify examples of natural language processing and its connection to mathematics and probability.
M.DSHS.35	Review ethical issues and the impact of machine learning and natural language processing.

Cluster	Utilize a coding language.
M.DSHS.36	Evaluate the appropriateness of programming languages and applications as they relate to data science.
M.DSHS.37	Select a programming language to explore, display, and analyze data.
M.DSHS.38	Identify types of information that can be stored as variables, classify variables, and utilize variables in programs that store data in appropriate ways (e.g., Booleans, characters, integers, floating points, strings).
M.DSHS.39	Interpret relational and logical expressions of level-appropriate complexity using comparison and Boolean operators.
M.DSHS.40	Create programming solutions by reusing existing code to perform analysis or retrieve data (e.g., libraries, APIs, publicly shared code).
M.DSHS.41	Write code or functions that can programmatically manipulate data sets (e.g., slice, merge, subset, sort, fit, summarize, analyze).

Cluster	Apply data science to a capstone project.
M.DSHS.42	Choose a problem or issue of interest. Throughout the program of study, research and use existing data set(s) to explore, visualize, analyze, and communicate findings to tell a data story.